Implementation of the Second-Order Nédélec Curl-Conforming Prismatic Element for Computational Electromagnetics

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Universidad Carlos III de Madrid
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- Definition of the element.
- Implementation of the element in HOFEM (*Higher Order Finite Element Method*).

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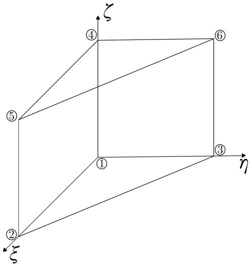

Space of Basis Functions

- Nédélec space of functions $\mathcal{R}^k$.

$$\mathcal{R}^k = \left\{\overline{u} \in (P_k)^n; \epsilon^k \overline{u} = 0\right\}$$

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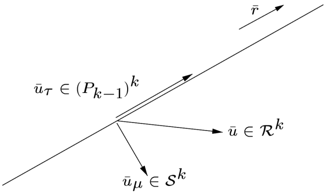

Space construction

- Tensor product between triangle and segment.

$$\mathcal{P}_{\boldsymbol{k}}^{\text{prism}} = (\mathcal{R}^k(\widehat{T}) \otimes \mathcal{P}_k(\widehat{I})) \times (\mathcal{P}_k(\widehat{T}) \otimes \mathcal{P}_{k-1}(\widehat{I}))$$

- Space dimension.

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Dimensions	$\mathcal{R}^k(\widehat{T})$	$\mathcal{P}_k(\widehat{I})$	$\mathcal{P}_k(\widehat{T})$	$\mathcal{P}_{k-1}(\widehat{I})$	Total
k	$(k+2)k$	$k+1$	$\frac{(k+1)(k+2)}{2}$	k	—
$k=1$	3	2	3	1	9
$k=2$	8	3	6	2	36
$k=3$	15	4	10	3	90
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Space definition

- First order.

$$\mathcal{P}_{\mathbf{1}}^{\text{prism}} \equiv \mathbf{N}_i(i=1,...,9) = \left\{ \begin{array}{c} a_1^{(i)} + a_2^{(i)}z + C^{(i)}y + D^{(i)}yz \\ b_1^{(i)} + b_2^{(i)}z - C^{(i)}x - D^{(i)}xz \\ c_1^{(i)} + c_2^{(i)}x + c_3^{(i)}y \end{array} \right\}$$

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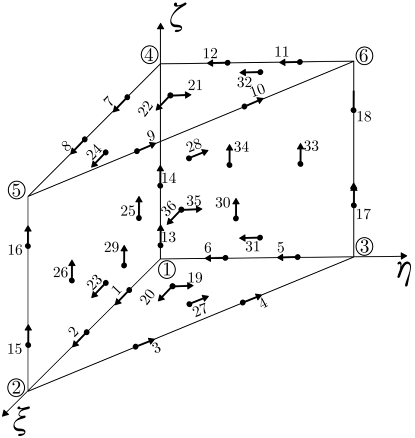
Master element



Other considerations (i)

- Discretization: choice of $\mathbf{q}$.

Dual basis

$$g_i(\mathbf{N_j}) = \delta_{ij}$$

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a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	$\ldots$
4	−6	−12	−16	24	48	12	−18	−36	$\ldots$
−2	6	2	8	−24	−8	−6	18	6	$\ldots$
0	0	2	0	0	−8	0	0	6	$\ldots$
0	0	4	0	0	−16	0	0	12	$\ldots$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$

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$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$

Other considerations (ii)

- Local definition of $\hat{\boldsymbol{\tau}}$, $\hat{\mathbf{n}}$, $\mathbf{q}$.
- Use of a master element:

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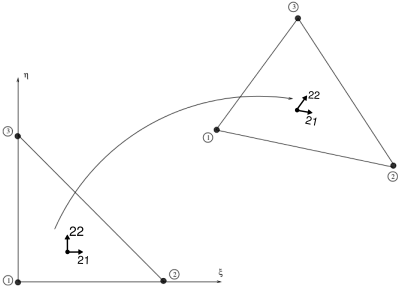

Other considerations (& iii)

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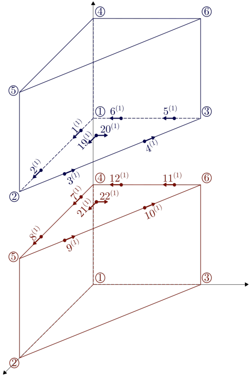

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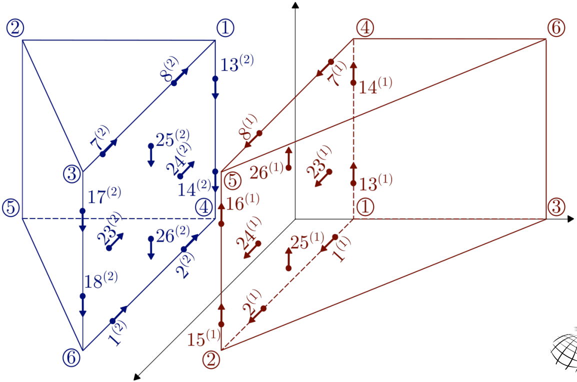
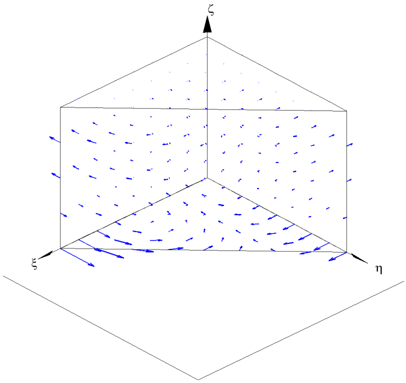
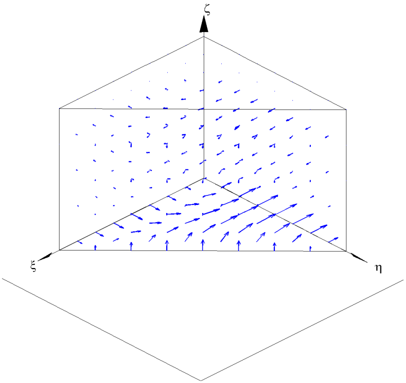


Basis functions on triangular faces

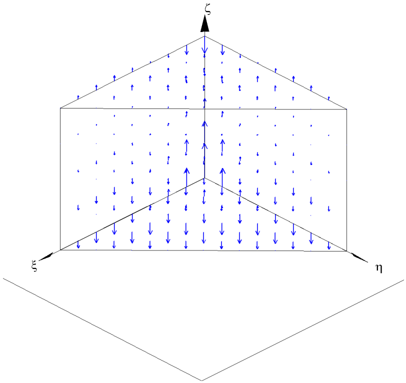


$\mathbf{N}_3$

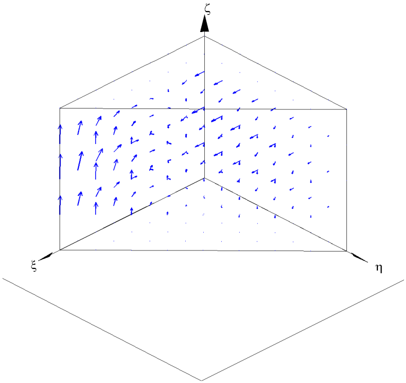


$\mathbf{N}_{19}$

Basis functions on quadrangular faces



$\mathbf{N}_{13}$



$\mathbf{N}_{24}$

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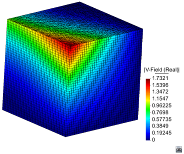
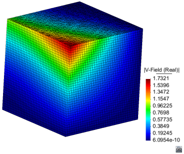
Method of Manufactured Solutions

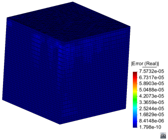
- $\nabla \times \frac{1}{\mathbf{f_r}} \nabla \times \mathbf{u} - k_0^2 \boldsymbol{g_r} \mathbf{u} = \boldsymbol{\Psi}$.
- HOFEM: Monomials $(xyz^2, -xz^2, xyz)$.



Analytical solution



Code solution



Error

Method of Manufactured Solutions

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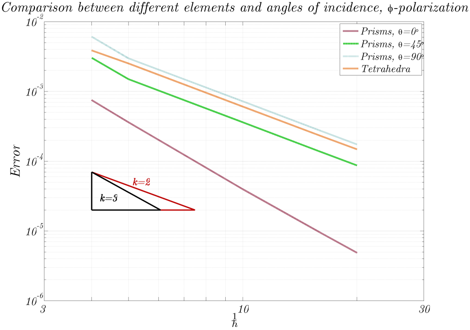
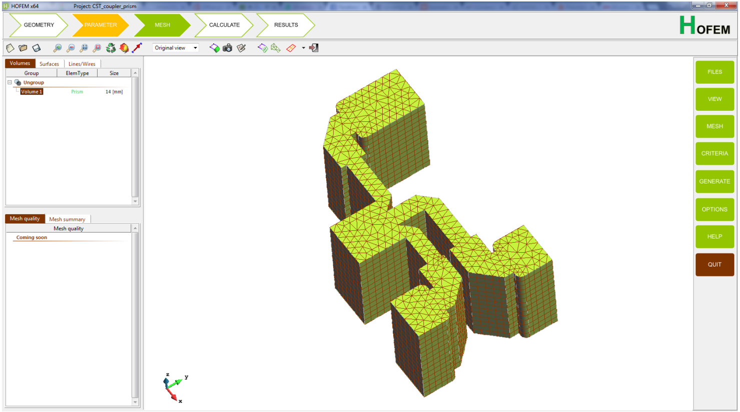
- HOFEM: Planewave.



Comparison (i)

- Structure to be simulated.



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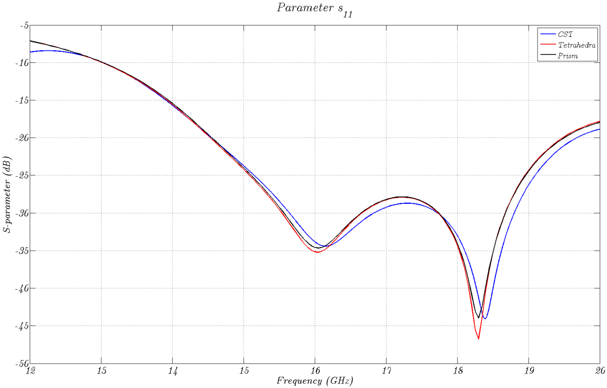

Comparison (i)

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Tetrahedra	22797	157240	2637.972
Prism	5220	83090	1061.215

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Comparison (& ii)

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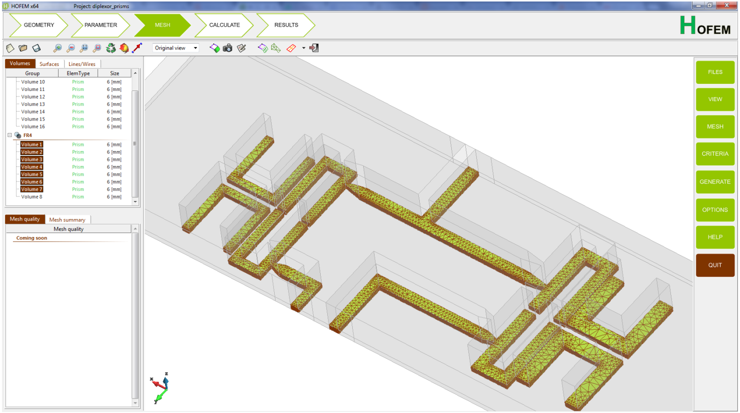

Comparison (& ii)

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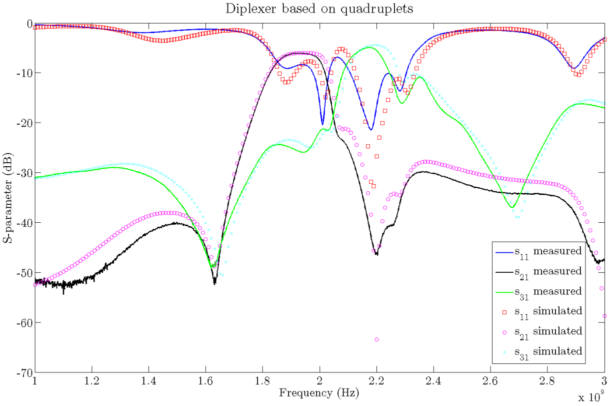

Comparison (& ii)

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—	Tetrahedra	Triangular prism	Finer mesh
Number of elements	14852	5444	138943
Number of unknowns	101828	91468	2054524
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Dudas y preguntas

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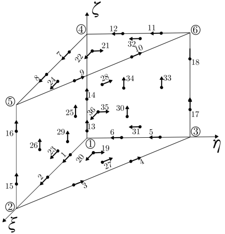

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