Analysis of Dispersion Error of Higher-Order Curl-Conforming Prismatic Finite Element

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Universidad Carlos III de Madrid

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NEMO 2017

18 May 2017

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Motivation

- FEM error investigated through 90's.
- Dispersion error.

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- Planar structures and waveguides.
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- D. García Doñoro, S. Ting, A.Amor Martín, L.E. García Castillo, "Analysis of Planar Microwave Devices using Higher Order Curl-Conforming Triangular Prismatic Finite Elements", Microwave and Optical Technology Letters, vol. 58, pp. 1794-1801, Jun. 2016.

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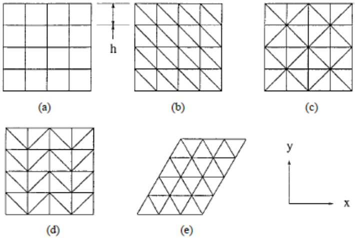
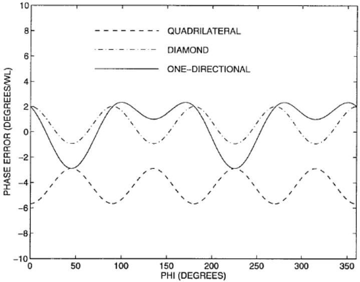
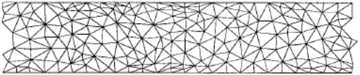
Antecedents (i)

- 1992: Lee.
- 1994: Warren, Scott.







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Antecedents (and ii)

- 1997: Wu, Lee.
- 2000: Ihlenburg, Babuska: $\mathcal{O}(h^{2p})$.
- 2003: Jin.

In short...

- Unstructured triangles in 2D.
- Unstructured tetrahedra in 3D.

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- Unstructured triangles in 2D.
- Unstructured tetrahedra in 3D.
- Structured tetrahedra and hexahedra is not encouraged.

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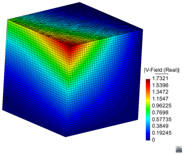
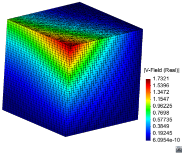
Verification: MMS

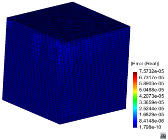
- $\nabla \times \frac{1}{\mathbf{f_r}} \nabla \times \mathbf{u} - k_0^2 \mathbf{g_r} \mathbf{u} = \mathbf{\Psi}$.
- HOFEM: Monomials $(xyz^2, -xz^2, xyz)$.



MMS solution



Code solution



Error

Verification: MMS

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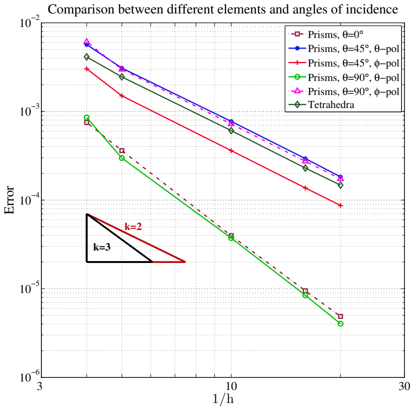
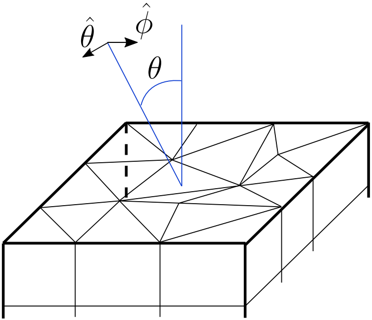
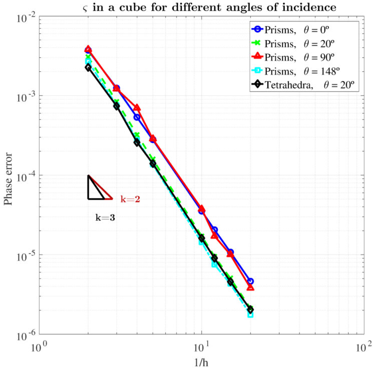
- HOFEM: Monomials $(xyz^2, -xz^2, xyz)$.
- HOFEM: Planewave.



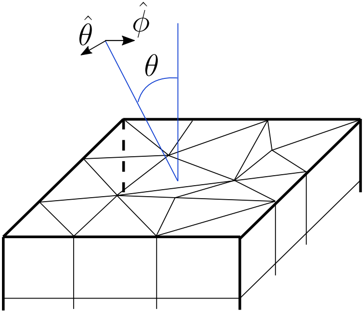
Phase error

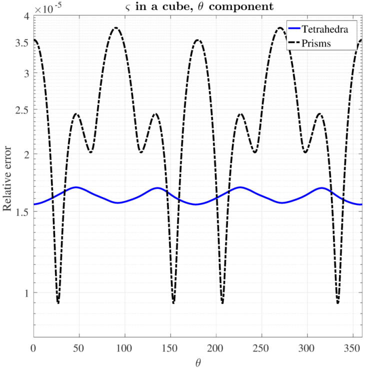




$$\varsigma = \frac{\int_\Omega |\measuredangle \mathbf{V}^\theta_{\mathrm{FEM}} - \measuredangle \mathbf{V}^\theta_{\mathrm{MMS}}| \, d\Omega}{\int_\Omega |\measuredangle \mathbf{V}^\theta_{\mathrm{MMS}}| \, d\Omega} \quad (1)$$

Phase error (and ii)

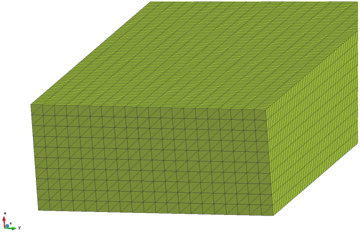


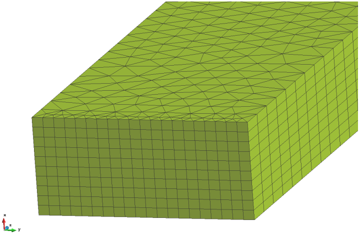


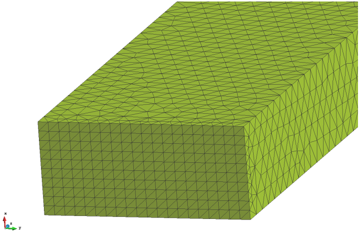
$$\varsigma = \frac{\int_\Omega |\measuredangle \mathbf{V}^\theta_{\text{FEM}} - \measuredangle \mathbf{V}^\theta_{\text{MMS}}| \, d\Omega}{\int_\Omega |\measuredangle \mathbf{V}^\theta_{\text{MMS}}| \, d\Omega} \quad (2)$$

Long waveguide (i)

- Length: 10λ.







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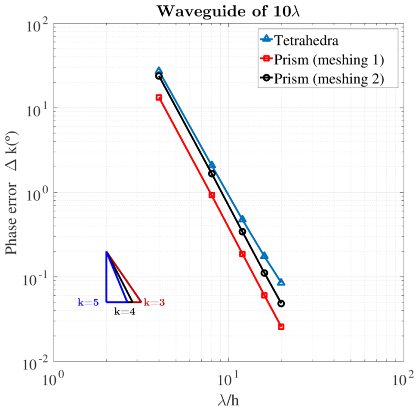

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Thank you for your attention!

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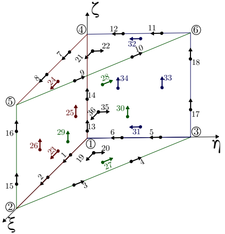

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