Towards a Scalable hp Adaptive Finite Element Code Based on a Non-Conformal Domain Decomposition Method

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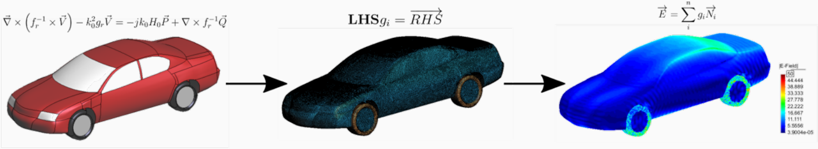
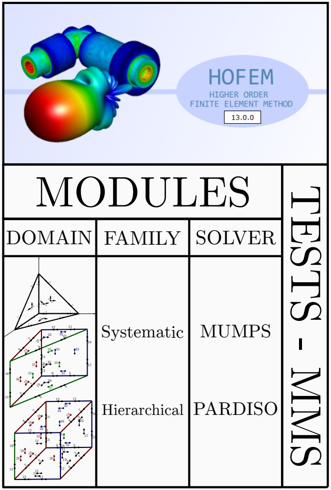
Introduction

- User-friendly.
- Based on GiD.



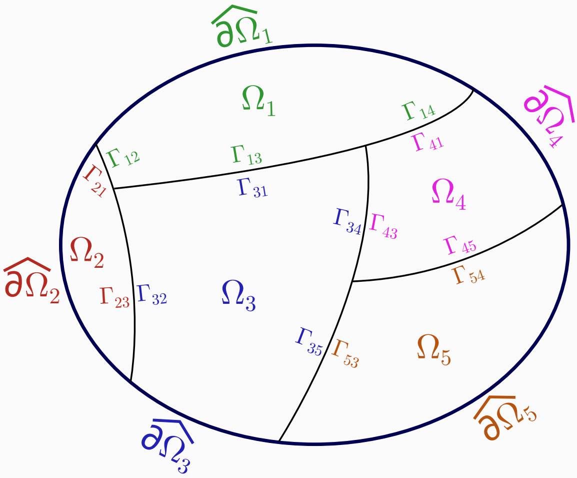
Intro: design by blocks



Intro: hp adaptivity

- *h* refinement.
- *p* refinement.
- *hp* refinement.

Intro: DDM



$\widehat{\partial\Omega_1}$
Ω_1
Γ_{14}
$\widehat{\partial\Omega_4}$
Γ_{41}
Γ_{12}
Γ_{13}
Γ_{21}
Γ_{31}
Ω_4
Γ_{34}
Γ_{43}
Γ_{45}
Ω_2
Γ_{54}
$\widehat{\partial\Omega_2}$
Γ_{23}
Γ_{32}
Ω_3
Γ_{35}
Γ_{53}
Ω_5
$\widehat{\partial\Omega_5}$
$\widehat{\partial\Omega_3}$

Intro: DDM

- Nonoverlapping, nonconformal and nonmatching.
- Hybrid meshes.
- Geometry aware optimization.
- *hp* adaptivity in 3D.

Formulation

Formulation: classic FEM

$$\boldsymbol{\nabla} \times \frac{1}{\mu_r}(\boldsymbol{\nabla} \times \mathbf{E}) - k_0^2 \varepsilon_r \mathbf{E} = \mathbf{O}$$

$$\begin{aligned} \widehat{\mathbf{n}} \times \mathbf{E} &= 0, \text{on } \Gamma_{\mathsf{D}} \\ \widehat{\mathbf{n}} \times \frac{1}{\mu_{ri}}(\boldsymbol{\nabla} \times \mathbf{E}) &= 0, \text{on } \Gamma_{\mathsf{N}} \\ \widehat{\mathbf{n}} \times \frac{1}{\mu_{ri}}(\boldsymbol{\nabla} \times \mathbf{E}) + jk_0 \widehat{\mathbf{n}} \times \widehat{\mathbf{n}} \times \mathbf{E} &= \boldsymbol{\Psi}, \text{on } \Gamma_{\mathsf{C}} \end{aligned}$$

Formulation: variational formulation

Find $\mathbf{E} \in \mathbf{W}$ such that

$$c_1(\mathbf{F},\mathbf{E}) - k_0^2\, c_2(\mathbf{F},\mathbf{E}) + \gamma\, c_3(\mathbf{F},\mathbf{E}) = l(\mathbf{F}), \quad \forall \mathbf{F} \in \mathbf{W}$$

$$c_1(\mathbf{F},\mathbf{E}) = \int_\Omega (\boldsymbol{\nabla} \times \mathbf{F}) \cdot (\frac{1}{\mu_r} \boldsymbol{\nabla} \times \mathbf{E})\, d\Omega$$

$$c_2(\mathbf{F},\mathbf{E}) = \int_\Omega \mathbf{F} \cdot \varepsilon_r \mathbf{E}\, d\Omega$$

$$c_3(\mathbf{F},\mathbf{E}) = \int_{\Gamma_\mathsf{C}} (\widehat{\mathbf{n}} \times \mathbf{F}) \cdot (\widehat{\mathbf{n}} \times \mathbf{E})\, d\Gamma_\mathsf{C}$$

$$l(\mathbf{F}) = \int_\Omega (\mathbf{F} \cdot \mathbf{O})\, d\Omega - \int_{\Gamma_\mathsf{C}} (\mathbf{F} \cdot \boldsymbol{\Psi})\, \Gamma_\mathsf{C}$$

$$\mathbf{W} := \{\mathbf{A} \in \mathbf{H}(\mathsf{curl}, \Omega),\ \widehat{\mathbf{n}} \times \mathbf{A} = 0 \text{ on } \Gamma_D\}$$

Formulation: FEM with DDM

$$\nabla \times \frac{1}{\mu_{ri}}(\nabla \times \mathbf{E}_i) - k_0^2 \varepsilon_{ri} \mathbf{E}_i = \mathbf{O}_i$$

$$\widehat{\mathbf{n}}_i \times \mathbf{E}_i = 0, \text{on } \Gamma_{i,\mathsf{D}}$$

$$\widehat{\mathbf{n}}_i \times \frac{1}{\mu_{ri}}(\nabla \times \mathbf{E}_i) = 0, \text{on } \Gamma_{i,\mathsf{N}}$$

$$\widehat{\mathbf{n}}_i \times \frac{1}{\mu_{ri}}(\nabla \times \mathbf{E}_i) + jk_0 \widehat{\mathbf{n}}_i \times \widehat{\mathbf{n}}_i \times \mathbf{E}_i = \mathbf{\Psi}_i, \text{on } \Gamma_{i,\mathsf{C}}$$

$$\widehat{\mathbf{n}}_i \times \mathbf{E}_i \times \widehat{\mathbf{n}}_i = \widehat{\mathbf{n}}_j \times \mathbf{E}_j \times \widehat{\mathbf{n}}_j, \text{on } \Gamma_{ij}$$

$$\widehat{\mathbf{n}}_i \times \frac{1}{\mu_{ri}}(\nabla \times \mathbf{E}_i) = -\widehat{\mathbf{n}}_j \times \frac{1}{\mu_{rj}}(\nabla \times \mathbf{E}_j), \text{on } \Gamma_{ij}$$

Formulation: cement variables and TC

$$\mathbf{e}_i = \widehat{\mathbf{n}}_i \times \mathbf{E}_i \times \widehat{\mathbf{n}}_i$$

$$\mathbf{j}_i = \frac{1}{k_0}\widehat{\mathbf{n}}_i \times \frac{1}{\mu_{ri}}(\boldsymbol{\nabla} \times \mathbf{E}_i)$$

$$\rho_i = \frac{1}{k_0}\nabla_\tau \cdot \mathbf{j}_i$$

$$(\alpha\mathcal{I}+\beta_i\mathcal{S}_{\mathsf{TE}})(\mathbf{e}_i) + (\mathcal{I} + \gamma_i\mathcal{S}_{\mathsf{TM}})(\mathbf{j}_i) = (\alpha\mathcal{I} + \beta_j\mathcal{S}_{\mathsf{TE}})(\mathbf{e}_j) - (\mathcal{I} + \gamma_j\mathcal{S}_{\mathsf{TM}})(\mathbf{j}_j)$$

$$\mathcal{S}_{\mathsf{TE}} = \boldsymbol{\nabla}_\tau \times \boldsymbol{\nabla}_\tau \times$$

$$\mathcal{S}_{\mathsf{TM}} = \nabla_\tau \nabla_\tau \cdot$$

Formulation: variational formulation with DDM (i)

Find $\mathbf{E}_i \in \mathbf{W}_i, \mathbf{j}_i \in \mathbf{X}_i, \rho_i \in Y_i$ such that

$$c_1(\mathbf{F}_i, \mathbf{E}_i) - k_0^2\, c_2(\mathbf{F}_i, \mathbf{E}_i) + jk_0\, c_{\tau,1}(\widehat{\mathbf{n}}_i \times \mathbf{F}_i, \widehat{\mathbf{n}}_i \times \mathbf{E}_i) = l(\mathbf{F}_i),\ \forall\, \mathbf{F}_i \in \mathbf{W}_i$$

$$\alpha\, c_{\tau,1}(\mathbf{l}_i, \mathbf{e}_i) + k_0\, c_{\tau,1}(\mathbf{l}_i, \mathbf{j}_i) + k_0^2 \gamma_i c_{\tau,1}(\mathbf{l}_i, \boldsymbol{\nabla}_\tau \rho_i) + \beta_i k_0\, c_{\tau,1}(\boldsymbol{\nabla}_\tau \times \mathbf{l}_i, \boldsymbol{\nabla}_\tau \times \mathbf{e}_i) = \alpha\, c_{\tau,1}(\mathbf{l}_i, \mathbf{e}_j) - k_0\, c_{\tau,1}(\mathbf{l}_i, \mathbf{j}_j) - k_0^2 \gamma_j c_{\tau,1}(\mathbf{l}_i, \boldsymbol{\nabla}_\tau \rho_j) + \beta_j k_0\, c_{\tau,1}(\boldsymbol{\nabla}_\tau \times \mathbf{l}_i, \boldsymbol{\nabla}_\tau \times \mathbf{e}_j),\ \forall\, \mathbf{l}_i \in \mathbf{X}_i$$

$$c_{\tau,1}(\boldsymbol{\nabla}_\tau \phi_i, \mathbf{j}_i) + k_0\, c_{\tau,2}(\phi_i, \rho_i) = 0,\ \forall\, \phi_i \in Y_i$$

$$\mathbf{F}_i \in \mathbf{W}_i := \mathbf{H}_0(\mathrm{curl}; \Omega_i),\ \mathbf{l}_i \in \mathbf{X}_i := \mathbf{H}_0(\mathrm{curl}_\tau; \Gamma_{ij}),$$
$$\phi_i \in Y_i := H_0^{-1/2}(\Gamma_{ij})$$

Formulation: variational formulation with DDM (& ii)

$$c_1(\mathbf{F}_i, \mathbf{E}_i) = \int_{\Omega_i} (\boldsymbol{\nabla} \times \mathbf{F}_i) \cdot \frac{1}{\mu_{ri}} (\boldsymbol{\nabla} \times \mathbf{E}_i) \, d\Omega_i$$

$$c_2(\mathbf{F}_i, \mathbf{E}_i) = \int_{\Omega_i} \mathbf{F}_i \cdot \varepsilon_{ri} \mathbf{E}_i \, d\Omega_i$$

$$c_{\tau,1}(\mathbf{l}_i, \mathbf{e}_j) = \int_{\Gamma_{ij}} (\mathbf{l}_i \cdot \mathbf{e}_j) \, d\Gamma_{ij}$$

$$c_{\tau,2}(\phi_i, \rho_j) = \int_{\Gamma_{ij}} (\phi_i \rho_j) \, d\Gamma_{ij}$$

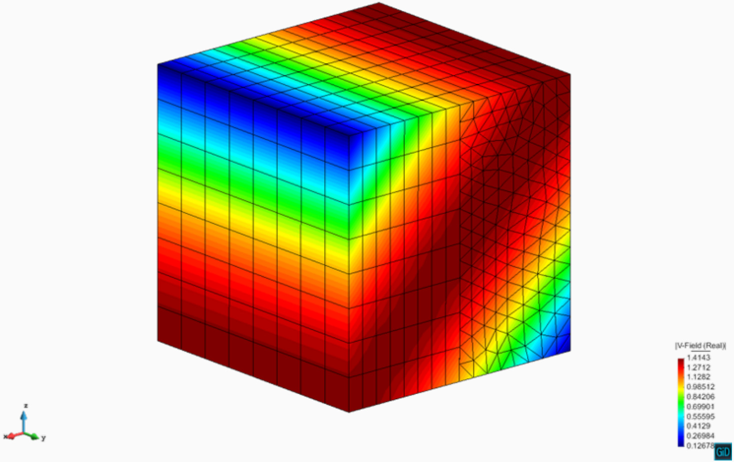
$$l(\mathbf{F}_i) = \int_{\Omega_i} (\mathbf{F}_i \cdot \mathbf{O}_i) \, d\Omega_i - \int_{\Gamma_{i,\mathsf{C}}} (\mathbf{F}_i \cdot \boldsymbol{\Psi}_i) \, \Gamma_{i,\mathsf{C}}$$

Verification

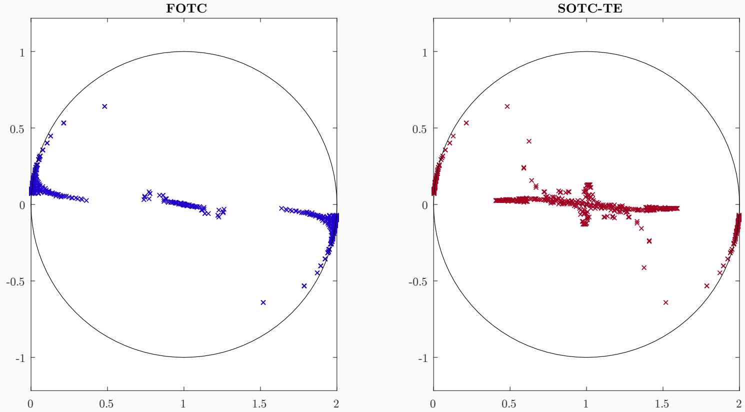
Verification: sources of error

- Introduction of domains: user-driven or ParMETIS.
- Shapes.
- Orders.
- Nonmatching interfaces.

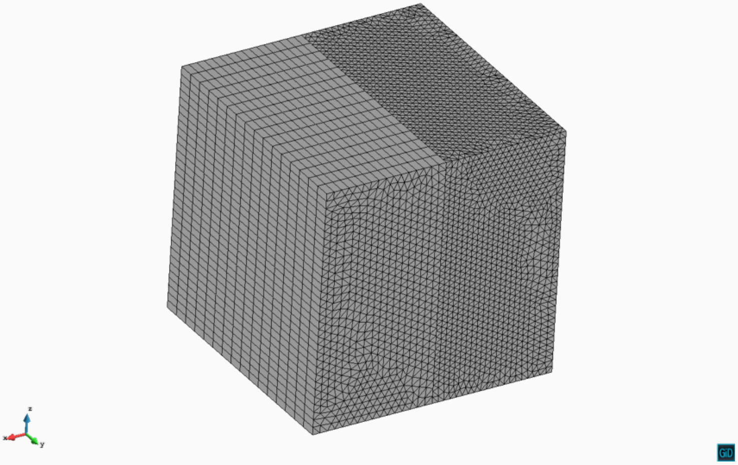
Verification: hexahedra with prisms



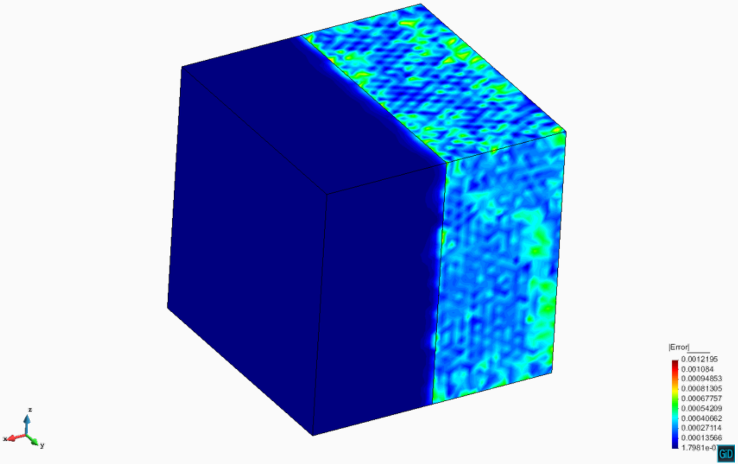
Verification: hexahedra with prisms



Verification: nonconformal in shape and order



Verification: nonconformal in shape and order



Numerical results

Results: setup

- Pre and postprocessing: GiD.
- Direct solver: MUMPS.
- Iterative solver: GCR through PETSC.
  - Residual: 10^{-6}.
- SOTC-TE.

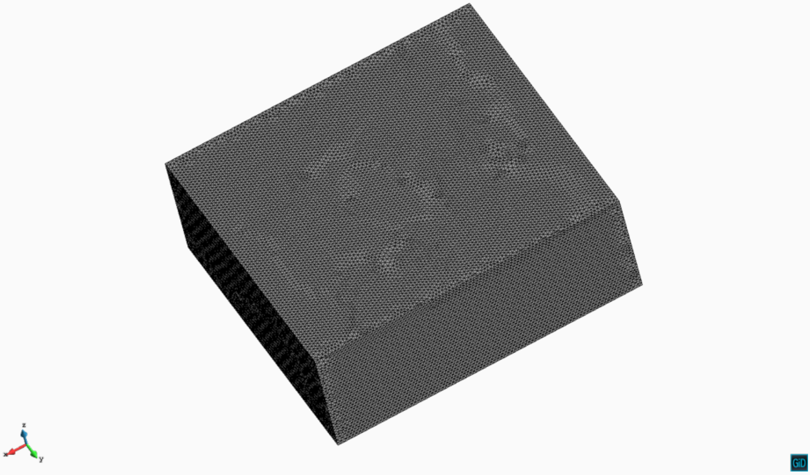
Results: 2D circular horn array

- 3x3 and 4x4 circular horns.
- Working frequency $f = 10\,\text{GHz}$
- Uniform excitation through WR-90 waveguides.
- Unstructured tetrahedra.

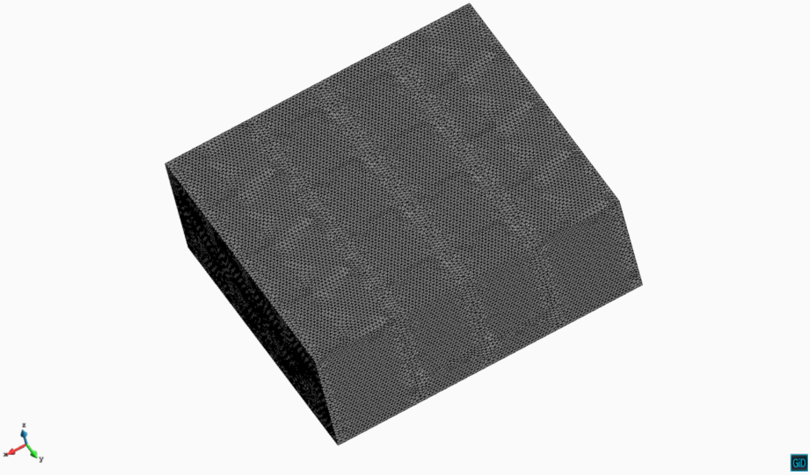
Results: 2D circular horn array

Case of study	Time (s)	Peak memory (Mb)	Unknowns	Surf. unkn	Iterations
3x3 No DDM	416	5380	1360188	—	—
3x3 DDM	463	3371	1398118	60144	62
4x4 No DDM	1579	12253	2261472	—	—
4x4 DDM	1191	5832	2368032	185856	73

Results: 2D circular horn array



Results: 2D circular horn array



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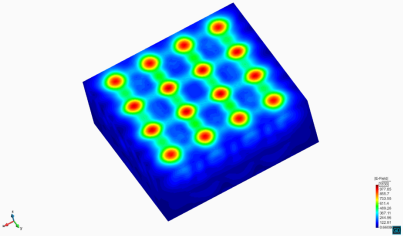
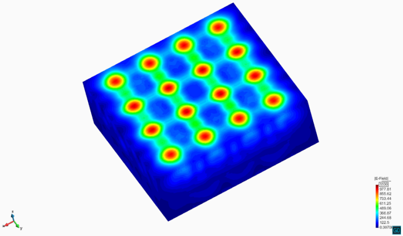
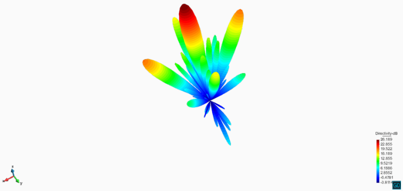
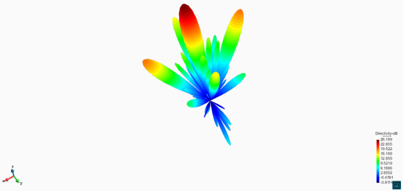




Results: 2D circular horn array





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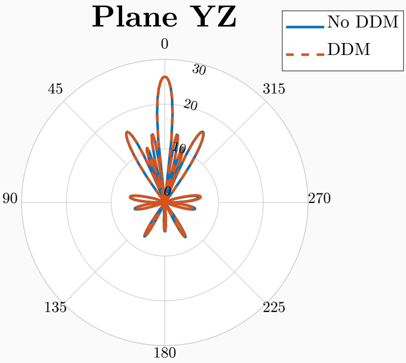
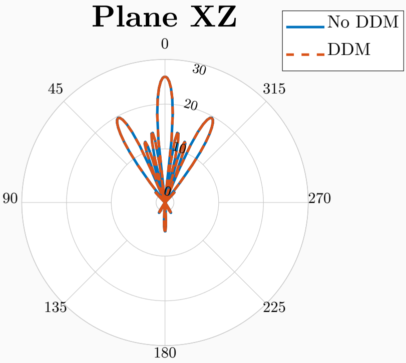


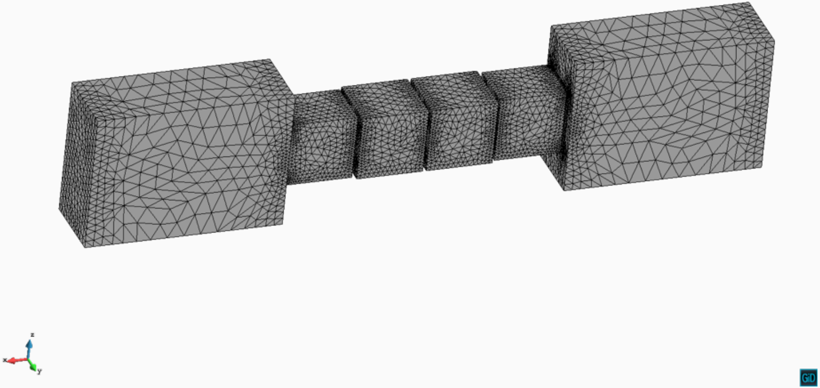


Results: X-band waveguide filter

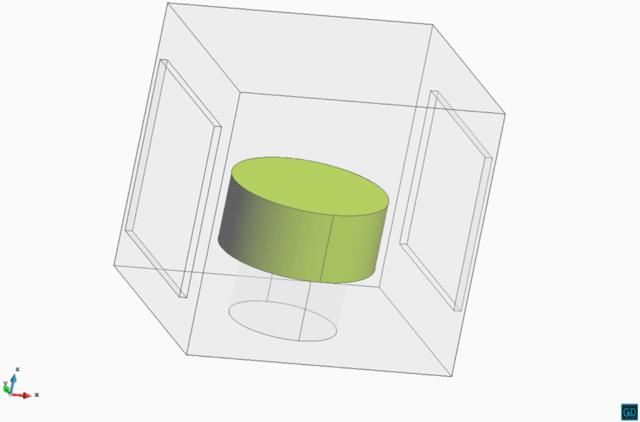
- WR-75 waveguide.
- X-band.
- 4 rectangular cavities:
  - Embedded resonator of $\varepsilon_r = 30$.
  - Support of $\varepsilon_r = 9$.
- 236690 unknowns for DDM, 412428 without DDM.

Alessandri, Ferdinando, et al. ”The electric-field integral-equation method for the analysis and design of a class of rectangular cavity filters loaded by dielectric and metallic cylindrical pucks.” *IEEE transactions on microwave theory and techniques*, 52.8 (2004): 1790-1797.

Results: X-band waveguide filter



Results: X-band waveguide filter



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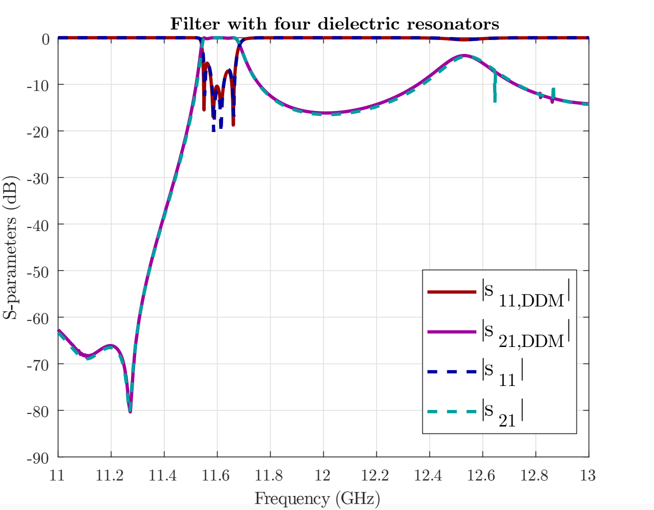
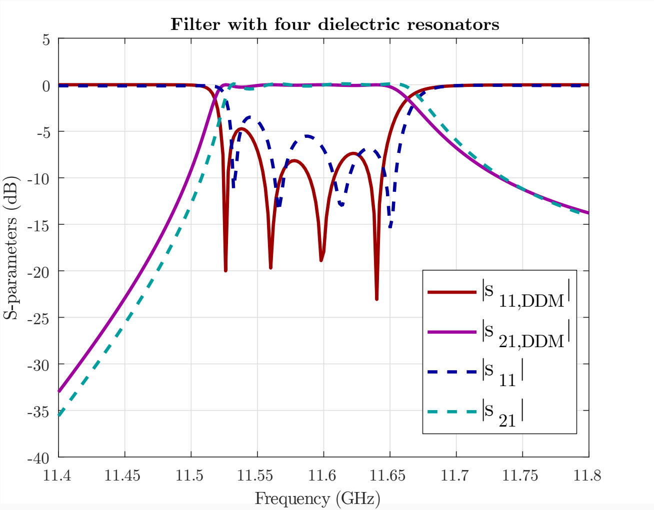


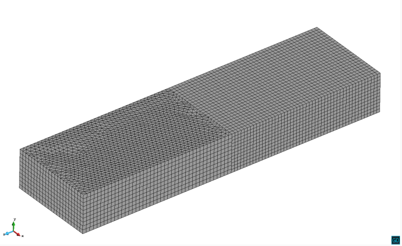
Results: X-band waveguide filter

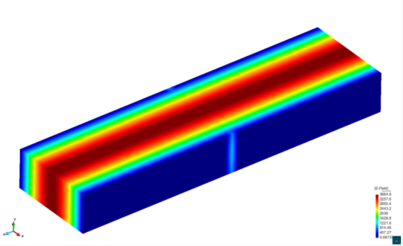


h adaptivity: nonconformal waveguide

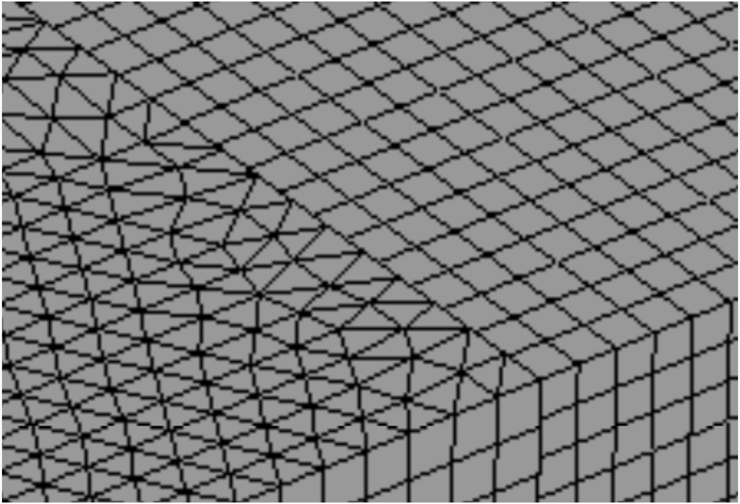
- WR-90 empty waveguide.
- Working frequency $f = 7.5$ GHz.
- Length of $10\lambda_g$.

h adaptivity: nonconformal waveguide

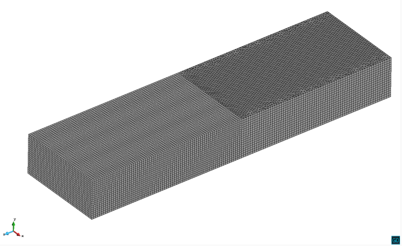


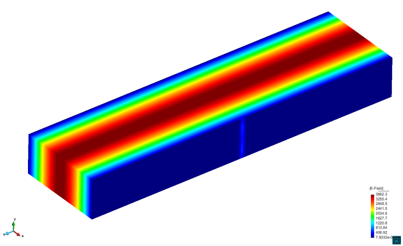


h adaptivity: nonconformal waveguide



h adaptivity: nonconformal waveguide





h adaptivity: steps

- Solve.
- Estimate.
- Mark.
- Refine.

h adaptivity: steps

$$\mathcal{R}_{\mathsf{vol},i}^{(m)} = \boldsymbol{\nabla} \times \mu_{ri}^{(-1)}(\boldsymbol{\nabla} \times \mathbf{E}_{i,\mathsf{FEM}}^{(m)}) - k_0^2 \varepsilon_{ri} \mathbf{E}_{i,\mathsf{FEM}}^{(m)} - \mathbf{O}_i,$$

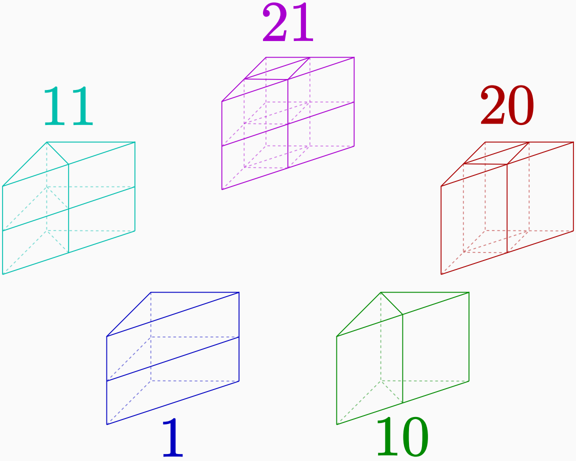
$$\mathcal{R}_{\mathsf{N}}^{(m)} = \widehat{\mathbf{n}}_i^{(m)} \times \mu_{ri}^{(-1)}(\boldsymbol{\nabla} \times \mathbf{E}_{i,\mathsf{FEM}}^{(m)}), \quad \text{on } \Gamma_{i,\mathsf{N}},$$

$$\mathcal{R}_{\mathsf{C}}^{(m)} = \widehat{\mathbf{n}}_i \times \mu_{ri}^{(-1)}(\boldsymbol{\nabla} \times \mathbf{E}_{i,\mathsf{FEM}}^{(m)}) + jk_0 \widehat{\mathbf{n}}_i^{(m)} \times \widehat{\mathbf{n}}_i^{(m)} \times (\boldsymbol{\Psi}_i - \mathbf{E}_{i,\mathsf{FEM}}^{(m)}), \quad \text{on } \Gamma_{i,\mathsf{C}}.$$

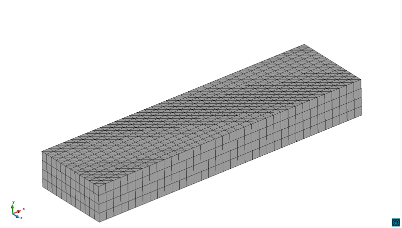
$$\mathcal{R}_{i,\mathsf{neigh}} \widehat{\mathbf{n}}_i^{(m)} = \widehat{\mathbf{n}}_i^{(m)} \times \mu_{ri}^{(-1)}(\boldsymbol{\nabla} \times \mathbf{E}_{i,\mathsf{FEM}}^{(m)}) + \widehat{\mathbf{n}}_i^{(n)} \times \mu_{ri}^{(-1)}(\boldsymbol{\nabla} \times \mathbf{E}_{i,\mathsf{FEM}}^{(n)}).$$

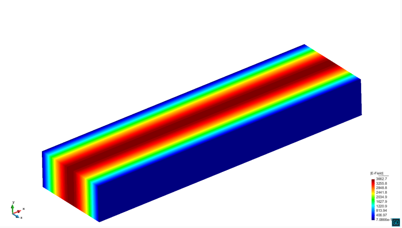
$$\mathcal{R}_{ij,\mathsf{DDM}}^{(m)} = \pi_\tau(\mathbf{E}_{i,\mathsf{FEM}}^{(m)}) + \pi_\tau^\times(\mu_{ri}^{(-1)} \boldsymbol{\nabla} \times \mathbf{E}_{i,\mathsf{FEM}}^{(m)}) - \pi_\tau(\mathbf{E}_{j,\mathsf{FEM}}^{(m)}) - \pi_\tau^\times(\mu_{rj}^{-1} \boldsymbol{\nabla} \times \mathbf{E}_{j,\mathsf{FEM}}^{(m)}).$$

h adaptivity: steps

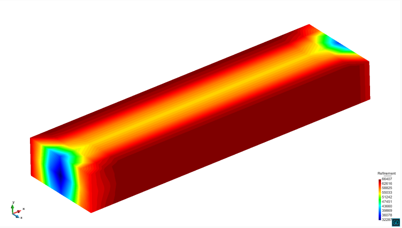


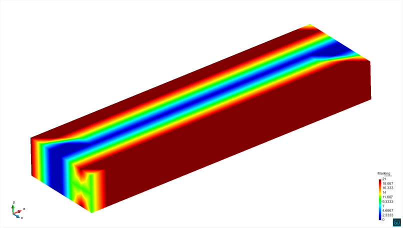
h adaptivity: steps



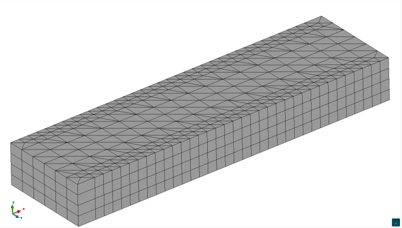


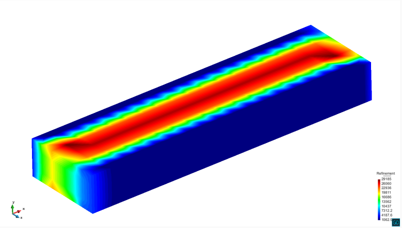
h adaptivity: steps



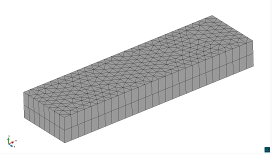


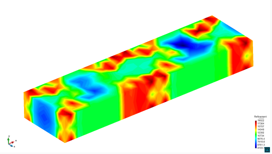
h adaptivity: steps

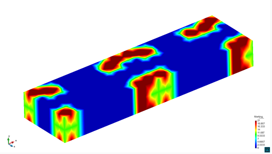




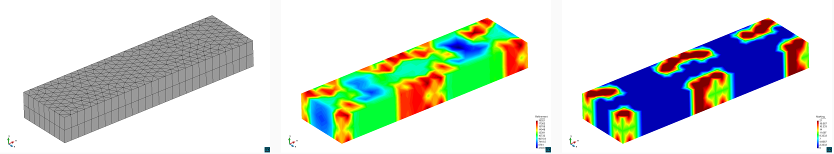
h adaptivity: unstructured mesh, no DDM



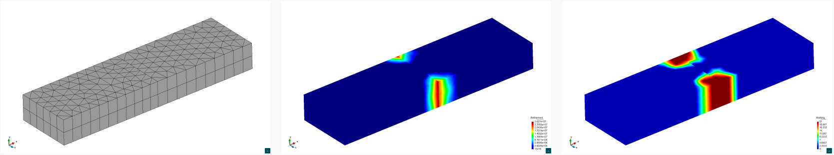




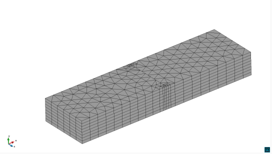
h adaptivity: unstructured mesh, conformal DDM

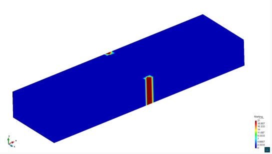


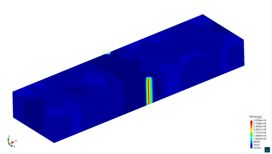
h adaptivity: unstructured mesh, nonconformal DDM



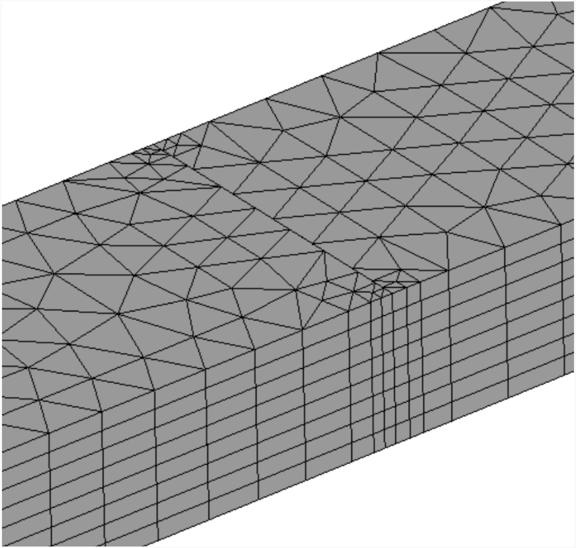
h adaptivity: unstructured mesh, nonconformal DDM







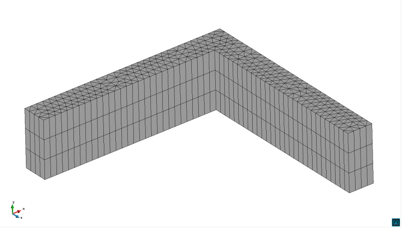
h adaptivity: unstructured mesh, nonconformal DDM

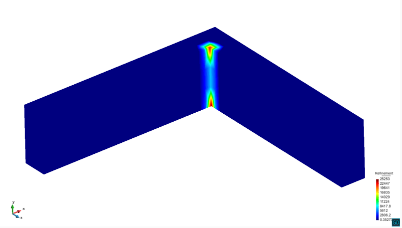


Bonus track: hp adaptivity

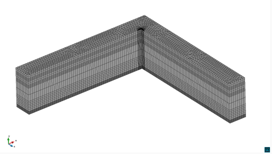
- Singularity through L-shape domain along E-plane.
- Working frequency $f = 7.5\,\text{GHz}$.
- WR-90 waveguide.

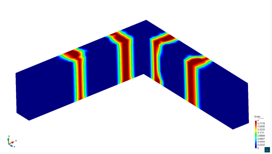
Bonus track: hp adaptivity

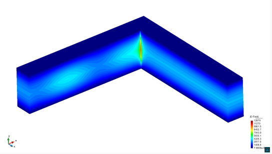




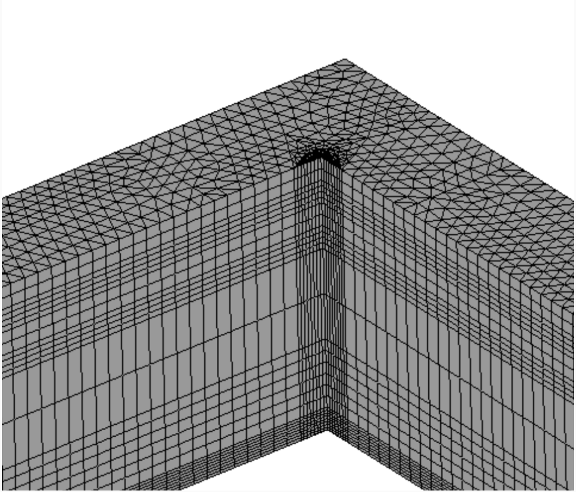
Bonus track: hp adaptivity



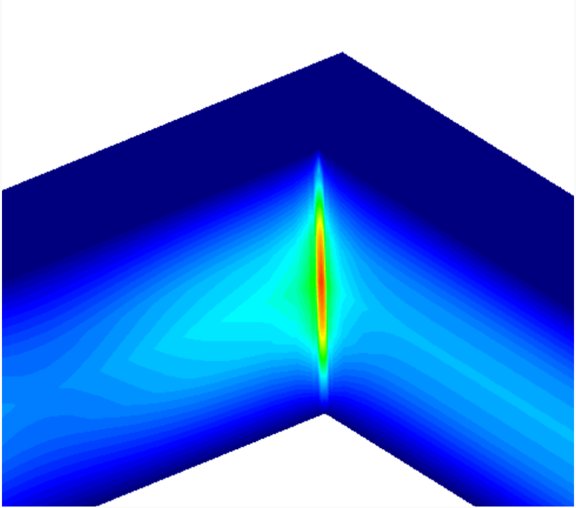




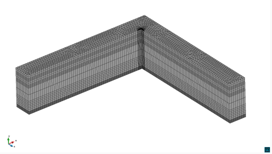
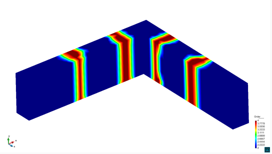
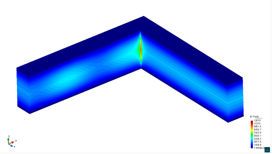
Bonus track: hp adaptivity



Bonus track: hp adaptivity



Bonus track: hp adaptivity

Conclusions

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- Introduction of DDM for hp refinement meshes.
- Geometry aware optimization with hybrid meshes.
- Best use of memory.
- Nonconformal study of adaptivity.

Thanks for your attention!

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