Three-level parallelization for Finite Element Code with Hybrid Meshes

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Introduction

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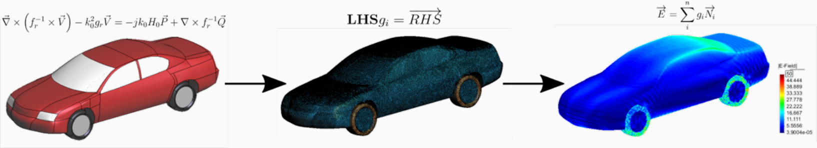
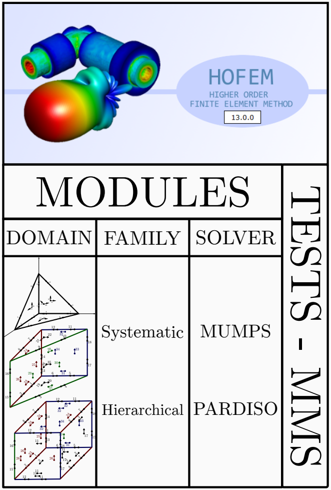
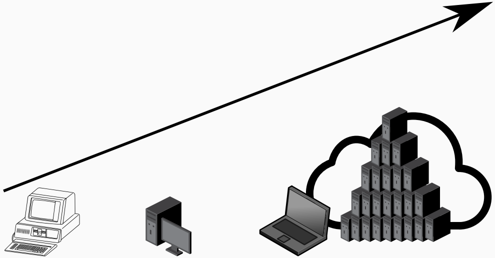
- User-friendly.
- Based on GiD.



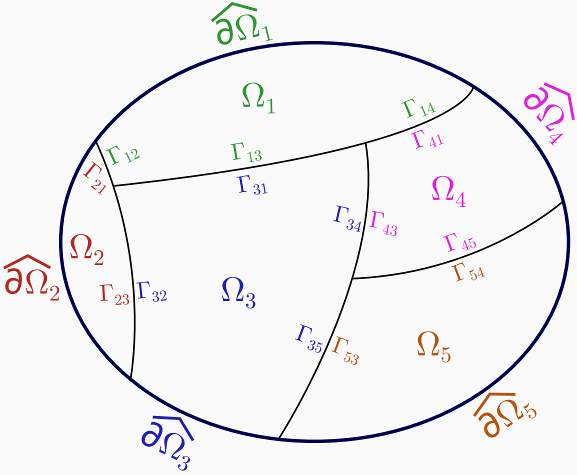
Intro: design by blocks



- Efficient use of HPC in electromagnetics.



- Memory limits (use of MUMPS).
- Domain Decomposition Method.
    - Memory.
    - Flexibility.



$\widehat{\partial\Omega_1}$
Ω_1
Γ_{14}
$\widehat{\partial\Omega_4}$
Γ_{41}
Γ_{12}
Γ_{13}
Γ_{21}
Γ_{31}
Ω_4
Γ_{34}
Γ_{43}
Ω_2
Γ_{45}
$\widehat{\partial\Omega_2}$
Γ_{54}
Γ_{23}
Γ_{32}
Ω_3
Γ_{35}
Γ_{53}
Ω_5
$\widehat{\partial\Omega_5}$
$\widehat{\partial\Omega_3}$

- Nonoverlapping, nonconformal and nonmatching.
- Hybrid meshes.
- Geometry aware optimization.
- *hp* adaptivity in 3D.

Formulation

Formulation: classic FEM

$$\nabla \times \frac{1}{\mu_r}(\nabla \times \mathbf{E}) - k_0^2 \varepsilon_r \mathbf{E} = \mathbf{O}$$

$$\widehat{\mathbf{n}} \times \mathbf{E} = 0, \text{on } \Gamma_{\mathsf{D}}$$

$$\widehat{\mathbf{n}} \times \frac{1}{\mu_{ri}}(\nabla \times \mathbf{E}) = 0, \text{on } \Gamma_{\mathsf{N}}$$

$$\widehat{\mathbf{n}} \times \frac{1}{\mu_{ri}}(\nabla \times \mathbf{E}) + jk_0 \widehat{\mathbf{n}} \times \widehat{\mathbf{n}} \times \mathbf{E} = \mathbf{\Psi}, \text{on } \Gamma_{\mathsf{C}}$$

Find $\mathbf{E} \in \mathbf{W}$ such that

$$c_1(\mathbf{F},\mathbf{E}) - k_0^2\, c_2(\mathbf{F},\mathbf{E}) + \gamma\, c_3(\mathbf{F},\mathbf{E}) = l(\mathbf{F}), \quad \forall \mathbf{F} \in \mathbf{W}$$

$$c_1(\mathbf{F},\mathbf{E}) = \int_\Omega (\boldsymbol{\nabla} \times \mathbf{F}) \cdot (\frac{1}{\mu_r} \boldsymbol{\nabla} \times \mathbf{E})\, d\Omega$$

$$c_2(\mathbf{F},\mathbf{E}) = \int_\Omega \mathbf{F} \cdot \varepsilon_r \mathbf{E}\, d\Omega$$

$$c_3(\mathbf{F},\mathbf{E}) = \int_{\Gamma_\mathsf{C}} (\widehat{\mathbf{n}} \times \mathbf{F}) \cdot (\widehat{\mathbf{n}} \times \mathbf{E})\, d\Gamma_\mathsf{C}$$

$$l(\mathbf{F}) = \int_\Omega (\mathbf{F} \cdot \mathbf{O})\, d\Omega - \int_{\Gamma_\mathsf{C}} (\mathbf{F} \cdot \boldsymbol{\Psi})\, \Gamma_\mathsf{C}$$

$$\mathbf{W} := \{\mathbf{A} \in \mathbf{H}(\mathsf{curl}, \Omega),\ \widehat{\mathbf{n}} \times \mathbf{A} = 0 \text{ on } \Gamma_D\}$$

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$$\nabla \times \frac{1}{\mu_{ri}}(\nabla \times \mathbf{E}_i) - k_0^2 \varepsilon_{ri} \mathbf{E}_i = \mathbf{O}_i$$

$$\begin{aligned}
\widehat{\mathbf{n}}_i \times \mathbf{E}_i &= 0, \text{on } \Gamma_{i,\mathsf{D}} \\
\widehat{\mathbf{n}}_i \times \frac{1}{\mu_{ri}}(\nabla \times \mathbf{E}_i) &= 0, \text{on } \Gamma_{i,\mathsf{N}} \\
\widehat{\mathbf{n}}_i \times \frac{1}{\mu_{ri}}(\nabla \times \mathbf{E}_i) + jk_0 \widehat{\mathbf{n}}_i \times \widehat{\mathbf{n}}_i \times \mathbf{E}_i &= \mathbf{\Psi}_i, \text{on } \Gamma_{i,\mathsf{C}} \\
\widehat{\mathbf{n}}_i \times \mathbf{E}_i \times \widehat{\mathbf{n}}_i &= \widehat{\mathbf{n}}_j \times \mathbf{E}_j \times \widehat{\mathbf{n}}_j, \text{on } \Gamma_{ij} \\
\widehat{\mathbf{n}}_i \times \frac{1}{\mu_{ri}}(\nabla \times \mathbf{E}_i) &= -\widehat{\mathbf{n}}_j \times \frac{1}{\mu_{rj}}(\nabla \times \mathbf{E}_j), \text{on } \Gamma_{ij}
\end{aligned}$$

$$\mathbf{e}_i = \widehat{\mathbf{n}}_i \times \mathbf{E}_i \times \widehat{\mathbf{n}}_i$$

$$\mathbf{j}_i = \frac{1}{k_0} \widehat{\mathbf{n}}_i \times \frac{1}{\mu_{ri}} (\boldsymbol{\nabla} \times \mathbf{E}_i)$$

$$\rho_i = \frac{1}{k_0} \nabla_\tau \cdot \mathbf{j}_i$$

$$\begin{aligned}(\alpha\mathcal{I}+\beta_i\mathcal{S}_{\mathsf{TE}})(\mathbf{e}_i) + (\mathcal{I} + \gamma_i\mathcal{S}_{\mathsf{TM}})(\mathbf{j}_i) = \\ (\alpha\mathcal{I} + \beta_j\mathcal{S}_{\mathsf{TE}})(\mathbf{e}_j) - (\mathcal{I} + \gamma_j\mathcal{S}_{\mathsf{TM}})(\mathbf{j}_j)\end{aligned}$$

$$\mathcal{S}_{\mathsf{TE}} = \boldsymbol{\nabla}_\tau \times \boldsymbol{\nabla}_\tau \times$$

$$\mathcal{S}_{\mathsf{TM}} = \nabla_\tau \nabla_\tau \cdot$$

Find $\mathbf{E}_i \in \mathbf{W}_i, \mathbf{j}_i \in \mathbf{X}_i, \rho_i \in Y_i$ such that

$$c_1(\mathbf{F}_i, \mathbf{E}_i) - k_0^2\, c_2(\mathbf{F}_i, \mathbf{E}_i) + jk_0\, c_{\tau,1}(\widehat{\mathbf{n}}_i \times \mathbf{F}_i, \widehat{\mathbf{n}}_i \times \mathbf{E}_i) = l(\mathbf{F}_i),\ \forall\, \mathbf{F}_i \in \mathbf{W}_i$$

$$\alpha\, c_{\tau,1}(\mathbf{l}_i, \mathbf{e}_i) + k_0\, c_{\tau,1}(\mathbf{l}_i, \mathbf{j}_i) + k_0^2 \gamma_i c_{\tau,1}(\mathbf{l}_i, \boldsymbol{\nabla}_\tau \rho_i) + \beta_i k_0\, c_{\tau,1}(\boldsymbol{\nabla}_\tau \times \mathbf{l}_i, \boldsymbol{\nabla}_\tau \times \mathbf{e}_i) = \alpha\, c_{\tau,1}(\mathbf{l}_i, \mathbf{e}_j) - k_0\, c_{\tau,1}(\mathbf{l}_i, \mathbf{j}_j) - k_0^2 \gamma_j c_{\tau,1}(\mathbf{l}_i, \boldsymbol{\nabla}_\tau \rho_j) + \beta_j k_0\, c_{\tau,1}(\boldsymbol{\nabla}_\tau \times \mathbf{l}_i, \boldsymbol{\nabla}_\tau \times \mathbf{e}_j),\ \forall\, \mathbf{l}_i \in \mathbf{X}_i$$

$$c_{\tau,1}(\boldsymbol{\nabla}_\tau \phi_i, \mathbf{j}_i) + k_0\, c_{\tau,2}(\phi_i, \rho_i) = 0,\ \forall\, \phi_i \in Y_i$$

$$\mathbf{F}_i \in \mathbf{W}_i := \mathbf{H}_0(\text{curl}; \Omega_i),\ \mathbf{l}_i \in \mathbf{X}_i := \mathbf{H}_0(\text{curl}_\tau; \Gamma_{ij}),\ \phi_i \in Y_i := H_0^{-1/2}(\Gamma_{ij})$$

$$c_1(\mathbf{F}_i, \mathbf{E}_i) = \int_{\Omega_i} (\boldsymbol{\nabla} \times \mathbf{F}_i) \cdot \frac{1}{\mu_{ri}} (\boldsymbol{\nabla} \times \mathbf{E}_i) \, d\Omega_i$$

$$c_2(\mathbf{F}_i, \mathbf{E}_i) = \int_{\Omega_i} \mathbf{F}_i \cdot \varepsilon_{ri} \mathbf{E}_i \, d\Omega_i$$

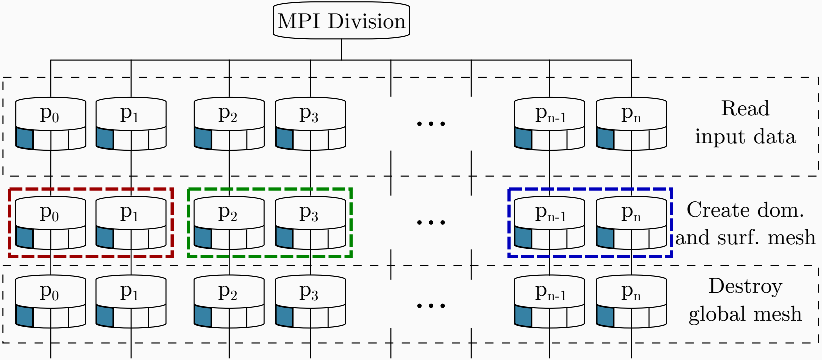
$$c_{\tau,1}(\mathbf{l}_i, \mathbf{e}_j) = \int_{\Gamma_{ij}} (\mathbf{l}_i \cdot \mathbf{e}_j) \, d\Gamma_{ij}$$

$$c_{\tau,2}(\phi_i, \rho_j) = \int_{\Gamma_{ij}} (\phi_i \rho_j) \, d\Gamma_{ij}$$

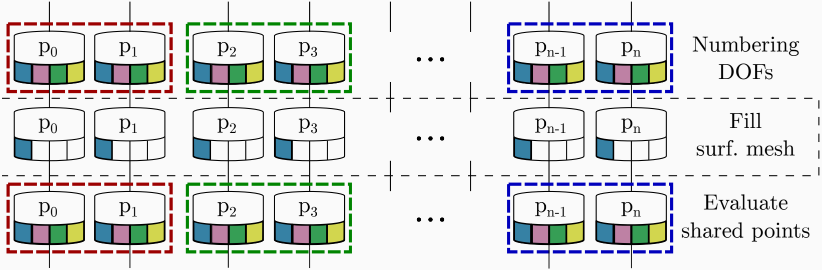
$$l(\mathbf{F}_i) = \int_{\Omega_i} (\mathbf{F}_i \cdot \mathbf{O}_i) \, d\Omega_i - \int_{\Gamma_{i,\mathsf{C}}} (\mathbf{F}_i \cdot \boldsymbol{\Psi}_i) \, \Gamma_{i,\mathsf{C}}$$

Implementation

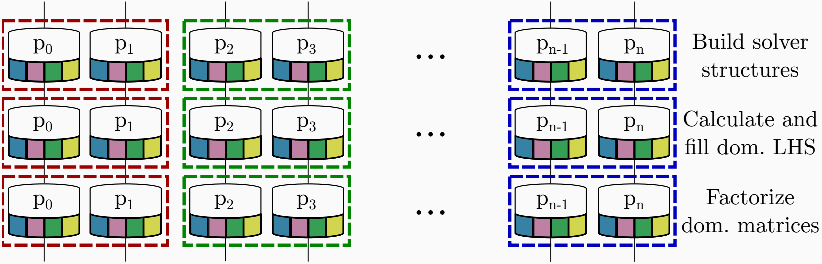
- Algorithm: DDM.
- Process: MPI.
- Thread: OpenMP.

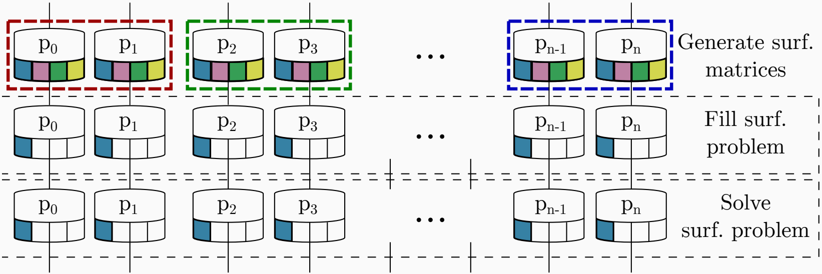

MPI Division
p0
p1
p2
p3
...
pn-1
pn
Read
input data
Create dom.
and surf. mesh
Destroy
global mesh

Implementation: workflow (ii)

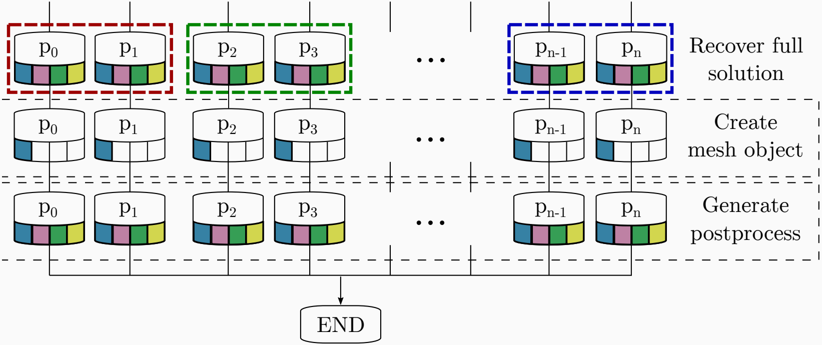


Implementation: workflow (iii)




p0
p1
p2
p3
...
pn-1
pn
Generate surf. matrices
Fill surf. problem
Solve surf. problem

Implementation: workflow (& v)



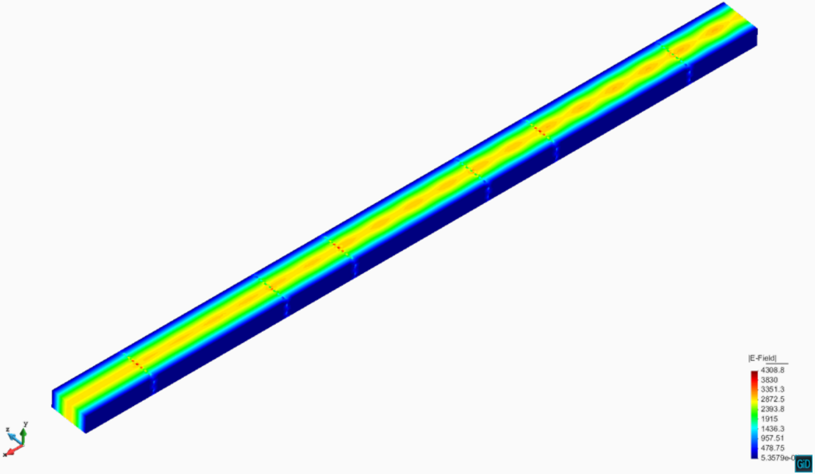
Numerical results

- Pre and postprocessing: GiD.
- Direct solver: MUMPS.
- Iterative solver: GCR through PETSC.
    - Residual: 10^{-6}.
- SOTC-TE.

- WR-90 empty waveguide.
- Working frequency $f = 10\,\mathrm{GHz}$.
- Length of $10\lambda_g$.
- 3 simulations:
  - Without DDM: 750688 unknowns.
  - With nonconformal DDM: tetrahedra, prisms and hexahedra: 512622 unknowns, 10 dom.


GiD

Results: long waveguide



Results: long waveguide

Case of study	Time (s)	Peak memory (Mb)
No DDM (MPI 1, OpenMP 8)	397.49	6867
No DDM (MPI 10, OpenMP 4)	58.475	2187
DDM (MPI 10, OpenMP 1)	228.48	1550
DDM (MPI 10, OpenMP 2)	184.02	1550
DDM (MPI 10, OpenMP 4)	163.18	1550

- WR-90 empty waveguide.
- Working frequency $f = 10\,\text{GHz}$.
- Length of $100\lambda_g$.
- 2 simulations:
    - Without DDM: 5042688 unknowns.
    - With *conformal* DDM, METIS: 26500 unknowns per dom., 200 dom.

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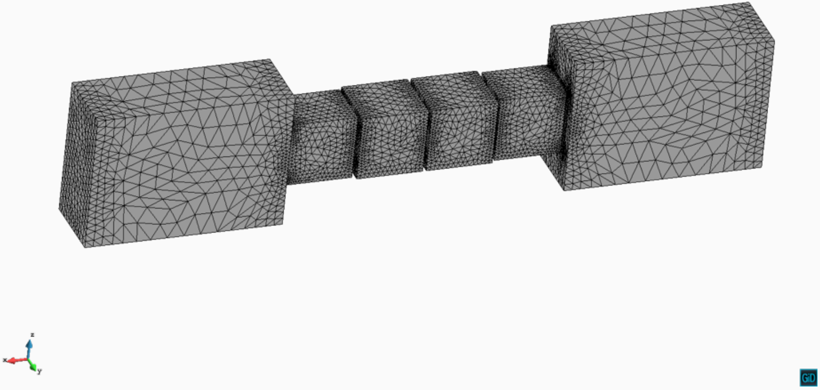
Case of study	Time (s)	Peak memory (Mb)
No DDM (MPI 12, OpenMP 4)	416.561	23060
DDM, 30 dom. (MPI 12, OpenMP 4)	1457.60	13467
DDM, 40 dom. (MPI 12, OpenMP 4)	1311.66	13244
DDM, 40 dom, direct (MPI 12, OpenMP 4)	1454.78	13399
DDM, 50 dom. (MPI 12, OpenMP 4)	1108.97	13329
DDM, 100 dom. (MPI 12, OpenMP 4)	1259.97	14217
DDM, 200 dom. (MPI 12, OpenMP 4)	2341.79	16076

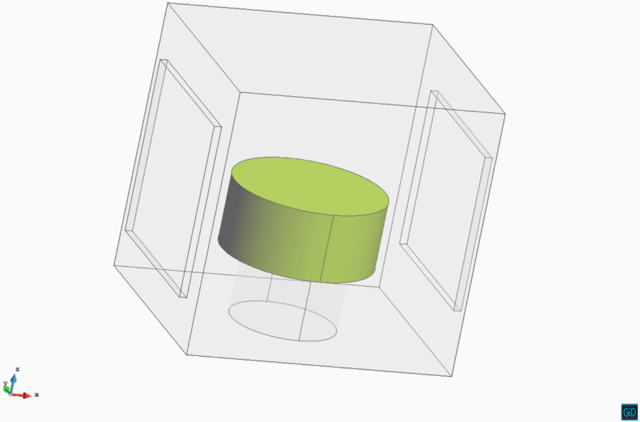
Results: X-band waveguide filter

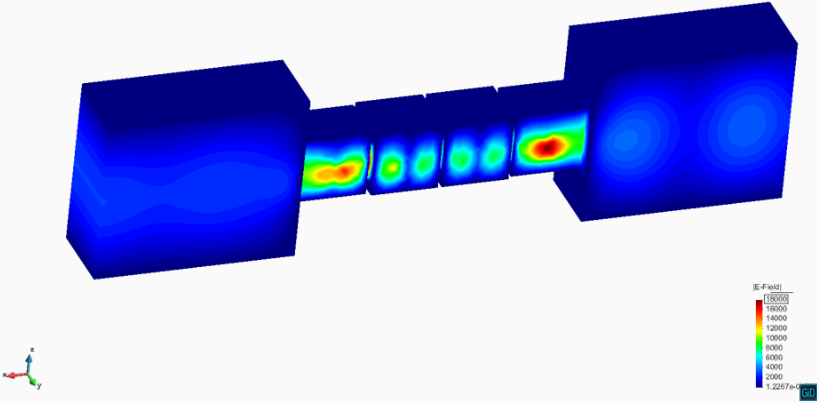
- WR-75 waveguide.
- X-band.
- 4 rectangular cavities:
  - Embedded resonator of $\varepsilon_r = 30$.
  - Support of $\varepsilon_r = 9$.
- 236690 unknowns for DDM, 412428 without DDM.

Alessandri, Ferdinando, et al. "The electric-field integral-equation method for the analysis and design of a class of rectangular cavity filters loaded by dielectric and metallic cylindrical pucks." *IEEE transactions on microwave theory and techniques*, 52.8 (2004): 1790-1797.

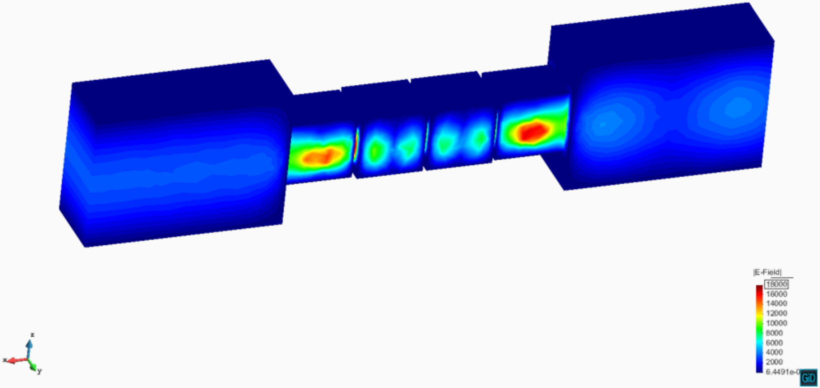
Results: X-band waveguide filter



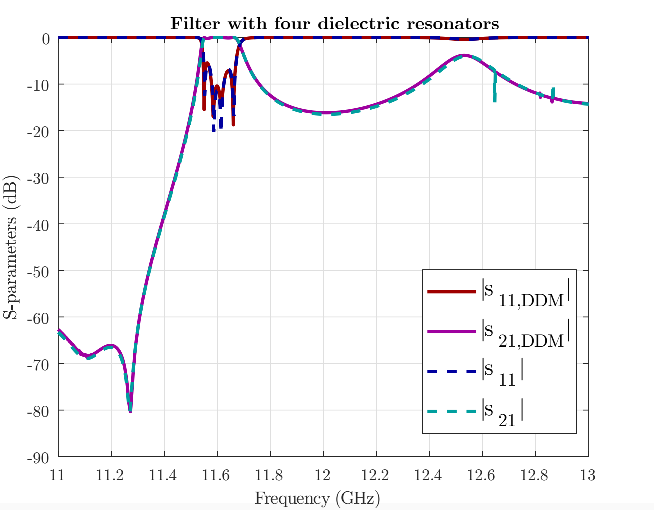

z
y
x
GiD


|E-Field|
18000
16000
14000
12000
10000
8000
6000
4000
2000
GiD
z
x
y

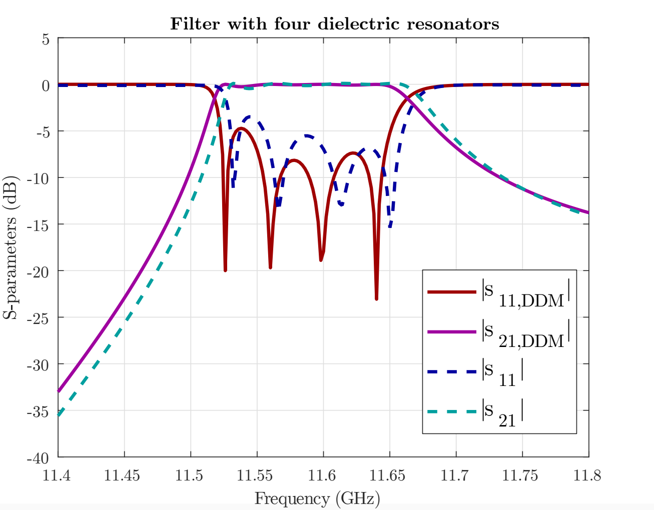
Results: X-band waveguide filter



Results: X-band waveguide filter



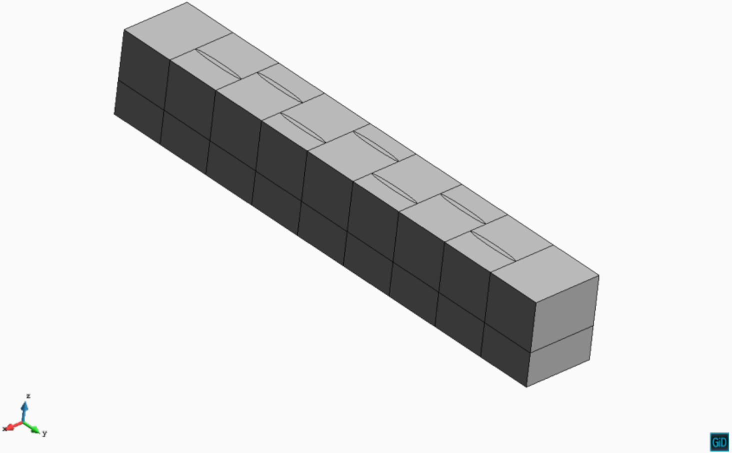
Results: X-band waveguide filter

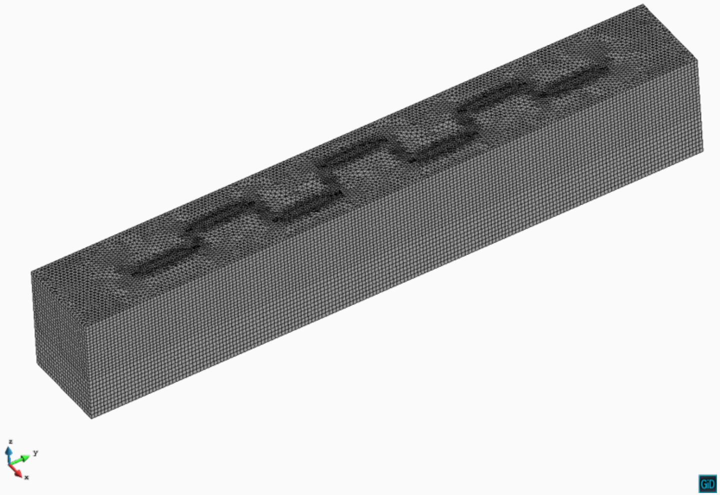


- Resonant SWA with length $4.5\lambda_g$.
- 7 elliptical slots.
- 9 subdomains.
- Working frequency: $f = 3.4045$ GHz.
- 611440 unknowns without DDM, 657256 with DDM.
- Systematic family of basis functions.
- Surface problem:
  - 141 iterations.
  - 61088 unknowns.

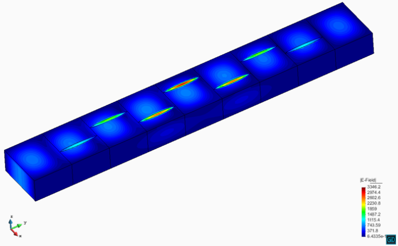
El Misilmani, Hilal M., Mohammed Al-Husseini, and Karim Y. Kabalan. ”Design of slotted waveguide antennas with low sidelobes for high power microwave applications.” *Progress In Electromagnetics Research* 56 (2015): 15-28.

Results: SWA

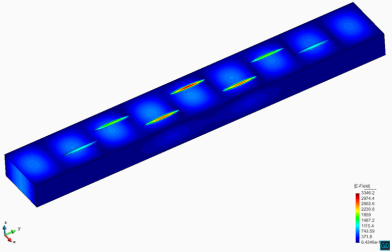



z
y
x
GiD

Results: SWA

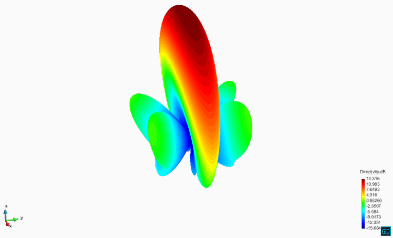


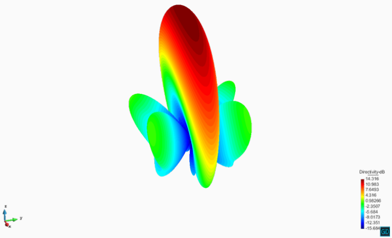
|E-Field|
3346.2
2974.4
2602.6
2230.8
1859
1487.2
1115.4
743.59
371.8
6.4335e-1
GiD

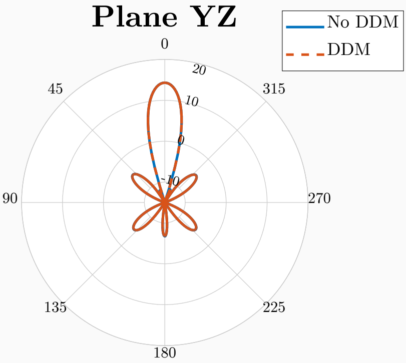


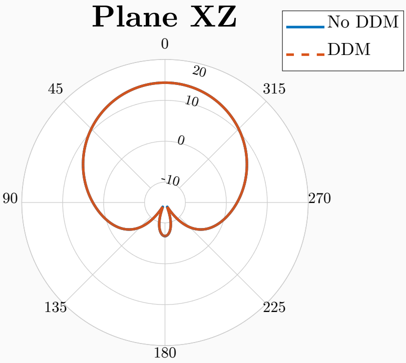
|E-Field|
3346.2
2974.4
2602.6
2230.8
1859
1487.2
1115.4
743.59
371.8
6.4346e-1
GiD

Results: SWA


Directivity-dB
14.316
10.983
7.6493
4.316
0.98266
-2.3507
-5.684
-9.0173
-12.351
-15.684
GiD

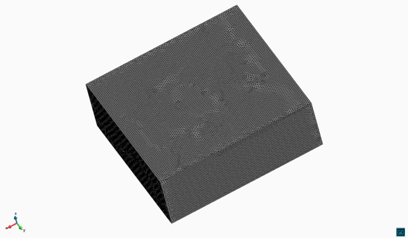

Directivity-dB
14.316
10.983
7.6493
4.316
0.98266
-2.3507
-5.684
-9.0173
-12.351
-15.684
GiD


Plane YZ
No DDM
DDM
0
45
315
90
270
135
225
180
20
10
0
-10

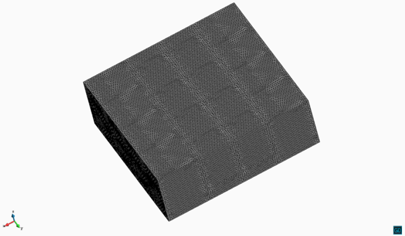

Plane XZ
No DDM
DDM
0
45
315
90
270
135
225
180
20
10
0
-10

- 4x4 circular horns.
- Working frequency $f = 10\,\mathrm{GHz}$
- Uniform excitation through WR-90 waveguides.
- 2261472 unknowns without DDM, 2368032 unknowns with DDM.
- Unstructured tetrahedra.
- Surface problem:
    - 73 iterations.
    - 185856 unknowns.

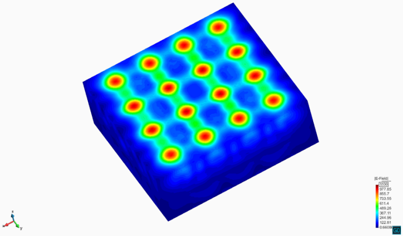
Results: 2D circular horn array

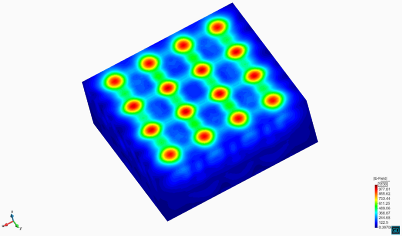


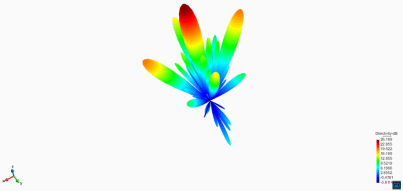
Results: 2D circular horn array

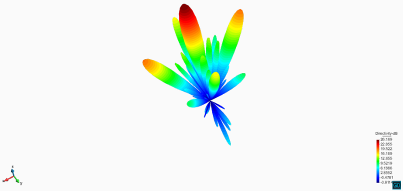


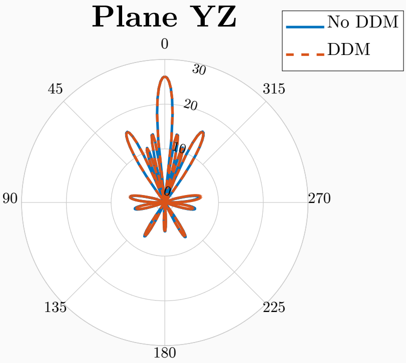
Results: 2D circular horn array

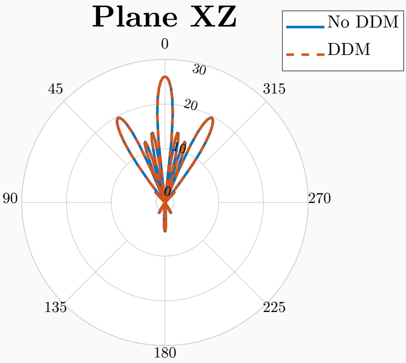





Directivity-dB
26.189
22.855
19.522
16.189
12.855
9.5219
6.1886
2.8552
-0.4781
-3.8114
GiD

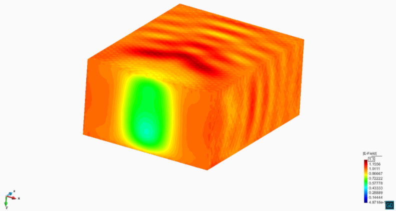

Directivity-dB
26.189
22.855
19.522
16.189
12.855
9.5219
6.1886
2.8552
-0.4781
-3.8114
GiD

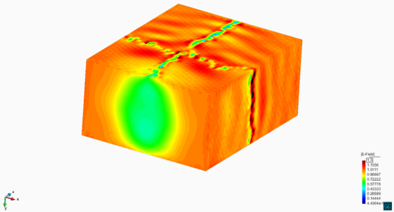

Plane YZ
No DDM
DDM
0
45
315
90
270
135
225
180
30
20
10
0

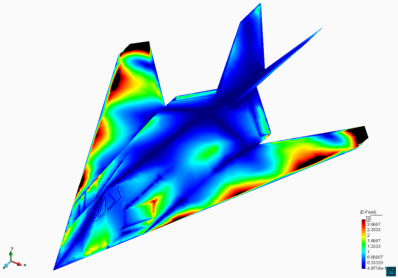

Plane XZ
No DDM
DDM
0
45
315
90
270
135
225
180
30
20
10
0

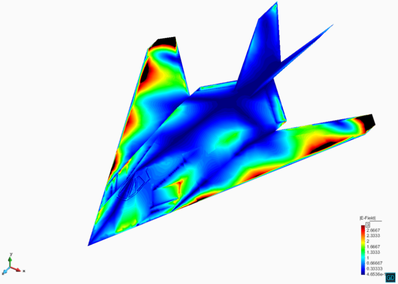
- Stealth design.
- Unstructured tetrahedra mesh.
- RCS for $f = 50$ MHz.
- Domains obtained through ParMETIS.
- Without DDM, 1011464 unknowns; with DDM, 1063208 unknowns.
- Surface problem (5 domains):
    - 360 iterations.
    - 68790 unknowns.

Results: RCS for F117

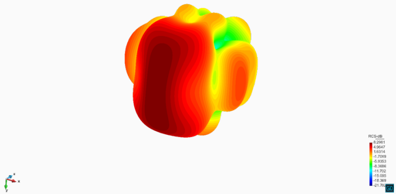


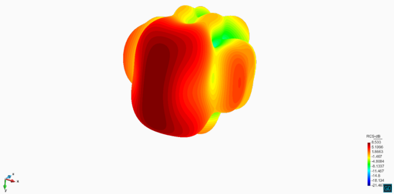


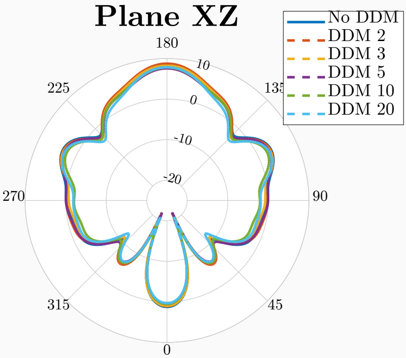

|E-Field|
3
2.6667
2.3333
2
1.6667
1.3333
1
0.66667
0.33333
4.8718e-1
GiD

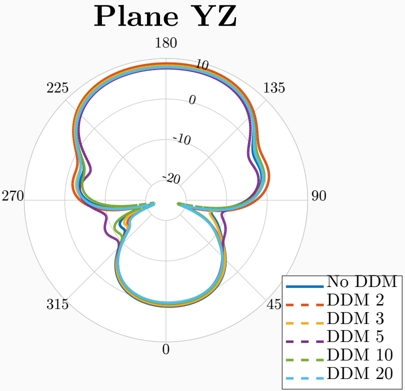

|E-Field|
3
2.6667
2.3333
2
1.6667
1.3333
1
0.66667
0.33333
4.6536e-1
GiD

Results: RCS for F117

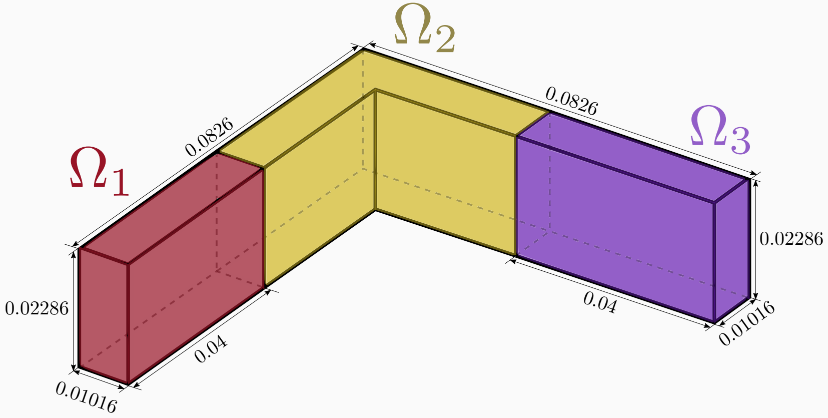




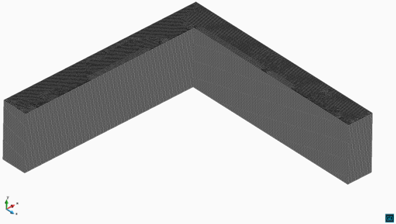

Plane XZ
180
10
0
-10
-20
225
135
270
90
315
45
0
No DDM
DDM 2
DDM 3
DDM 5
DDM 10
DDM 20

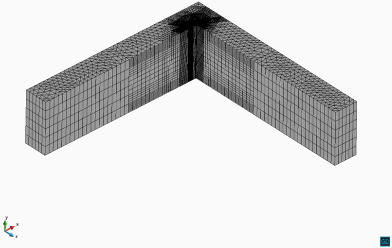

Plane YZ
180
10
0
-10
-20
225
135
270
90
315
45
0
No DDM
DDM 2
DDM 3
DDM 5
DDM 10
DDM 20

- Singularity through l-shape domain

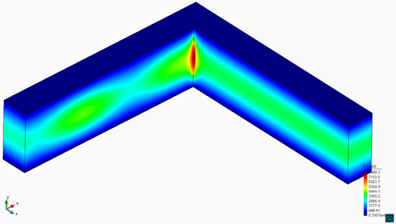


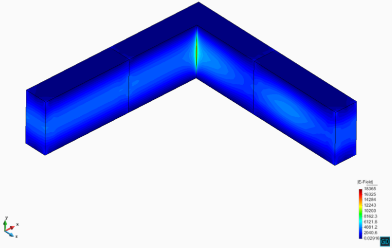
Bonus track: h adaptivity



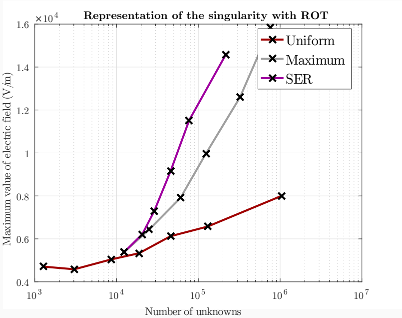


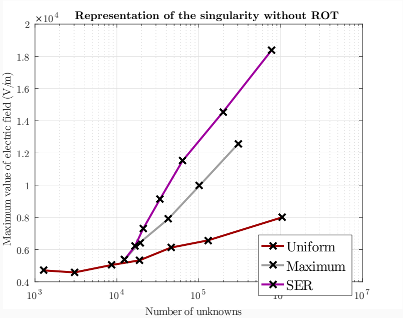
Bonus track: h adaptivity

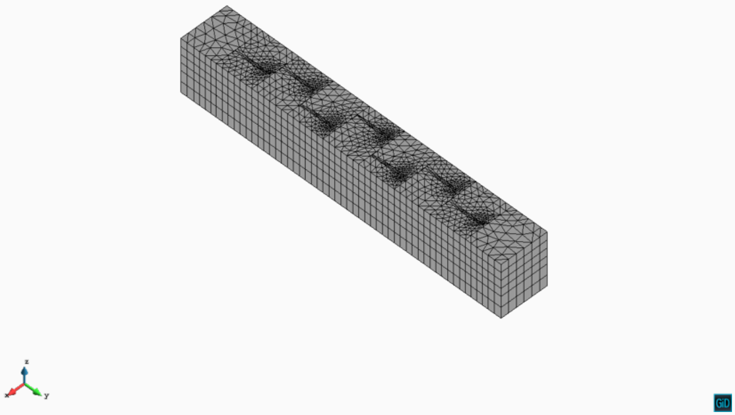




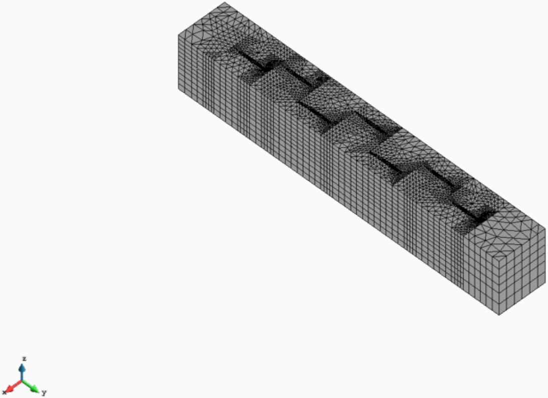
Bonus track: h adaptivity



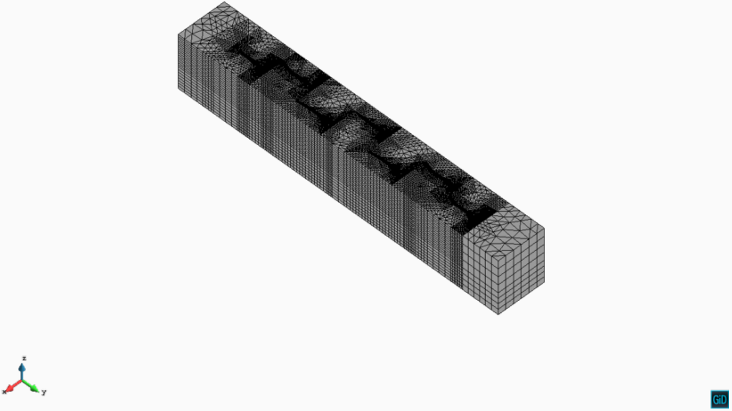



z
x
y
GiD

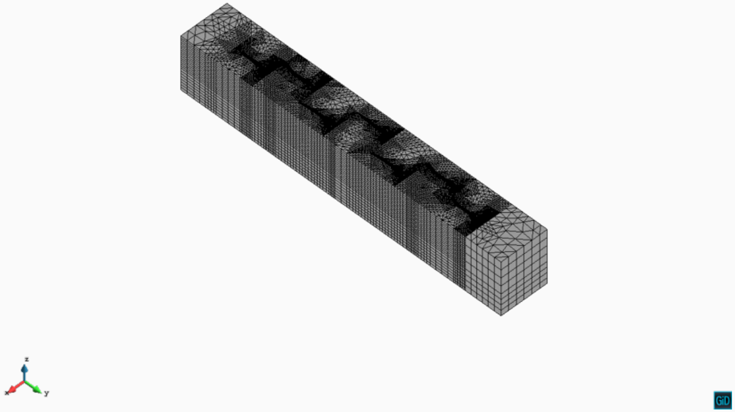
Bonus track: h adaptivity for SWA



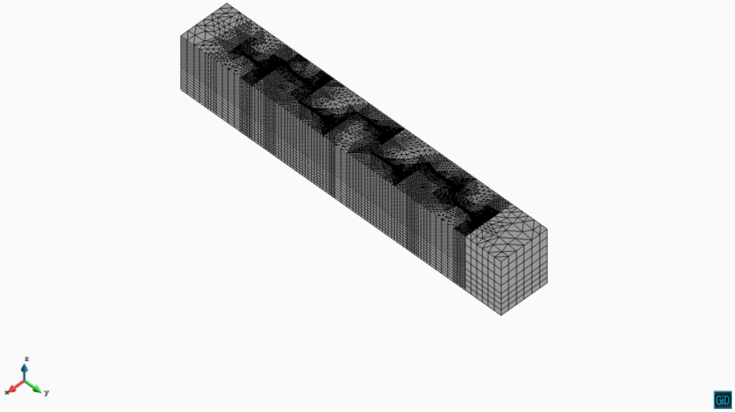
Bonus track: h adaptivity for SWA



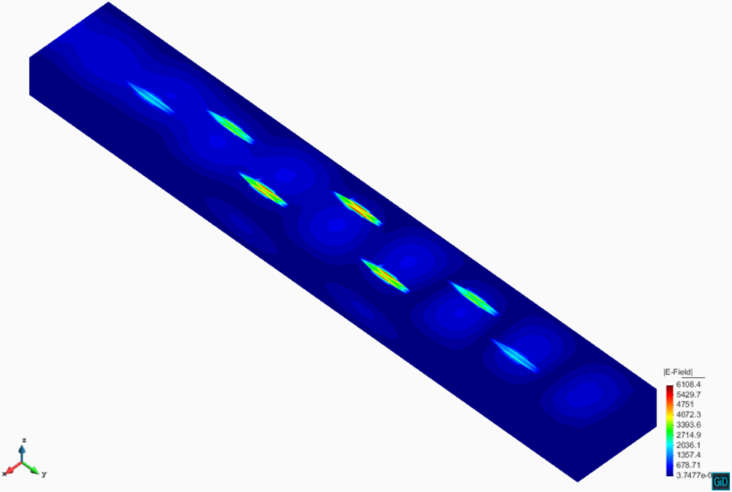
Bonus track: h adaptivity for SWA



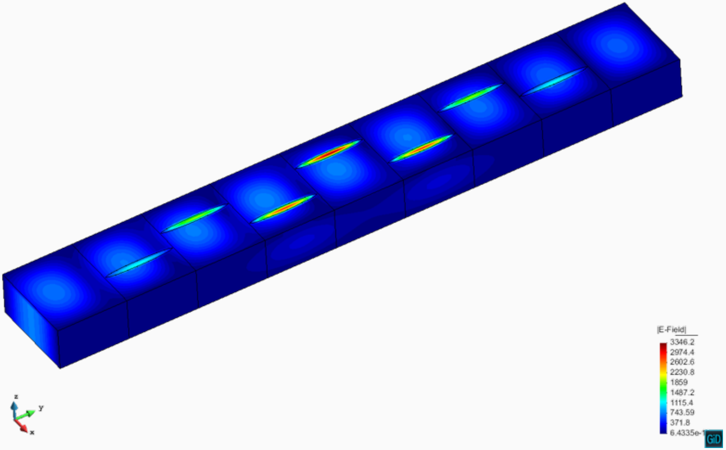
Bonus track: h adaptivity for SWA



Bonus track: h adaptivity for SWA

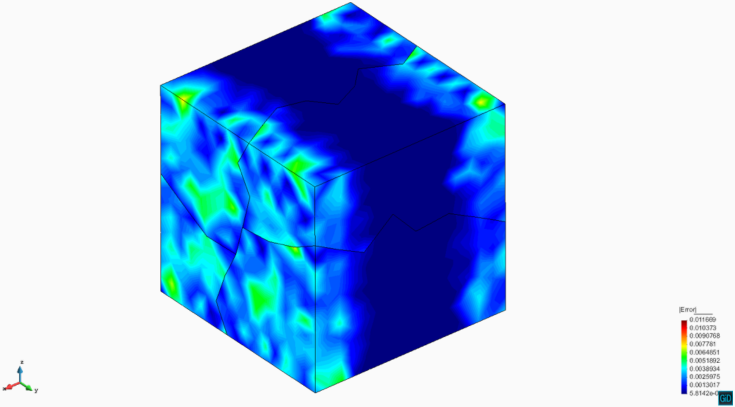


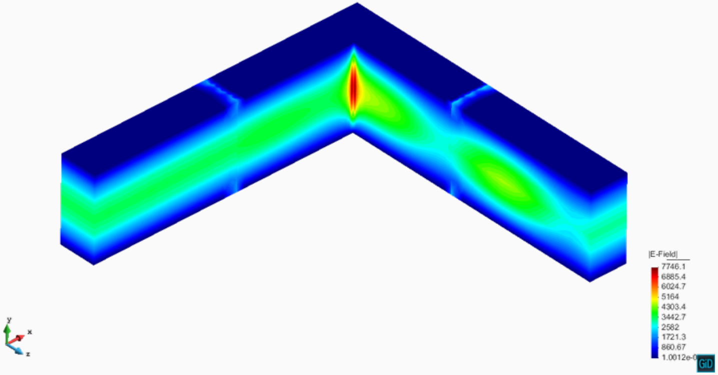
Bonus track: h adaptivity for SWA



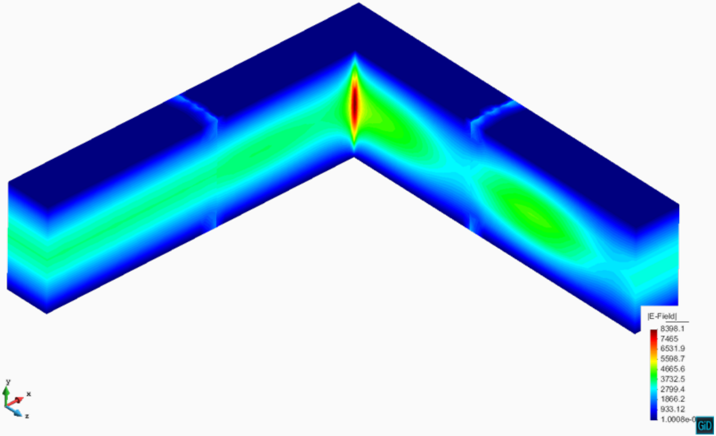
Hidden gem: hp adaptivity

- 134578 unknowns for h adaptivity.
- 67512 unknowns for hp adaptivity.




|E-Field|
7746.1
6885.4
6024.7
5164
4303.4
3442.7
2582
1721.3
860.67
GiD

Hidden gem: hp adaptivity



Conclusions

Conclusions

- Introduction of DDM as third level.
- Geometry aware optimization with hybrid meshes.
- Best use of memory for large scale problems.
- Controlled loss of accuracy.
- Flexibility.

Thanks for your attention!

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